ASSIGNMENT 2 – Business Statistics (201-934-DW)
Due Date: Friday December 4th 2009 4pm

ASSIGNMENT GUIDELINES

- Make sure you show all your work
- If you are using the z-table make sure you justify its use (parent population is normal OR $n \geq 30$)
- Remember to properly choose the use of the z-table or the t-table
- If it is not specified, you can use either the classical or the p-value approach to hypothesis testing

1- The heights of kindergarten children are approximately normally distributed with $\mu = 39$ inches and $\sigma = 2$ inches.
a- If an individual kindergarten child is selected at random what is the probability that he or she has a height between 38 and 40 inches?
b- A classroom of 30 children is used as a sample. What is the probability that the class mean height (average height of 30 students) is between 38 and 40 inches?
c- If an individual kindergarten child is selected at random, what is the probability that he or she is taller than 40 inches?
d- A classroom of 30 of these kindergarten children is used as a sample. What is the probability that the class mean is greater than 40 inches?

2- The lengths of 200 fish caught in Cayuga Lake had a mean of 14.3 inches. The population standard deviation is $\sigma = 2.5$ inches. Find the 90% confidence interval for the population mean length.

3- A high-tech company wants to estimate the mean number of years of college education its employees have completed. The standard deviation for the number of years of college is 1.0. How large a sample needs to be taken to estimate μ to within 0.5 years at 99% confidence?

4- Consider the hypothesis test where the hypotheses are Ho: $\mu \geq 26.4$ and Ha: $\mu < 26.4$. A random sample of size 64 is selected and yields a sample mean of 23.6. If significance level is 5% would you reject Ho?

5- From a population with unknown mean μ and standard deviation $\sigma = 5$, a sample of $n = 100$ is selected and the sample mean 40.6 is found.
a- Determine the 95% confidence interval for μ
b- Complete the hypothesis test for the claim $\mu = 40$ using the p-value approach and significance level $\alpha = 0.05$

6- The standard deviation of a normally distributed population is equal to 10. A sample of 25 is selected and its mean is found to be 95.
a- Find an 80% confidence interval for μ
b- If the sample size were 100, what would be the 80% confidence interval?
c- If the sample size were 25 but the standard deviation were 5, what would be the 80% confidence interval?

7- The null hypothesis Ho: $\mu = 48$ was tested against the alternative hypothesis Ha: $\mu \neq 48$. A sample of 75 resulted in a calculated p-value of 0.102. If $\sigma = 3.5$, find the value of the sample mean.

8- Homes in a nearby town have a mean value of $88 950. It is assumed that homes on the outskirts of the town have a higher value. To test this theory, a random sample of 12 homes from the outskirts of the town is selected. Their mean valuation is $92 460 and their sample standard deviation is $5200. Assuming home prices are normally distributed conduct a hypothesis test at 5% significance.

9- A student group maintains that the average student must travel for at least 25 minutes to reach their college everyday. The college admissions office obtained a random sample of 31 travel times from students. The sample had a mean of 19.4 minutes and a standard deviation of 9.6 minutes. Does the admissions office have sufficient evidence to reject the students' claim? Use significance 0.01.

10- To test the claim "the mean weight of adult men exceeds 160lb", 24 men were randomly selected and weighed yielding the following results:

173	178	145	146	157	175	173	137	152	171	163	170
135	159	199	203	172	162	156	141	172	188	160	165

Assuming that men's weights are normally distributed, test the claim at 0.01 significance.

11- In a sample of 60 randomly selected students, only 22 favoured the amount budgeted for next year's intramural and interscholastic sports. Construct the 99% confidence interval for the proportion of all students who support the proposed budget amount.

12- State the null hypothesis and the alternative hypothesis for the following scenarios:
a- More than 60% of all students at our college work part time jobs during the academic year
b- The probability of our team winning tonight is less than 0.50
c- No more than one-third of cigarette smokers are interested in quitting
d- At least 50% of all parents believe in spanking their children when appropriate.
e- A majority of voters will vote for the school budget this year
f- At least three-quarters of the trees in our city were seriously damaged by the ice storm
g- The results show the coin was not tossed fairly

13- A politician claims that she will receive 60% of the vote in an upcoming election. The results of a properly designed random sample of 100 voters showed that 50 of those sampled will vote for her. Is it likely that her assertion is correct at the 0.05 significance level?

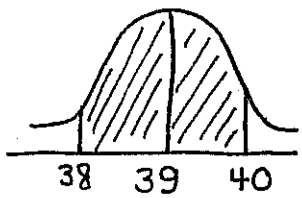
14- An insurance company states that 90% of its claims are settled within 30 days. A consumer group selected a random sample of 75 of the company's claims to test this statement. If the consumer group found that 55 of the claims were settled within 30 days, do they have sufficient reasons to support their contention that fewer than 90% of the claims are settled within 30 days? Use significance 5% and the p-value approach to test the claims.

15- A bank randomly selected 250 checking account customers and found that 110 of them also had savings accounts at this same bank. Construct a 95% confidence interval for the true proportion of checking account customers who also have savings accounts.

ASSIGNMENT 2
201-934-DW
SOLUTIONS
FALL 2009

(1) $\mu = 39 \quad \sigma = 2 \quad n = 30$

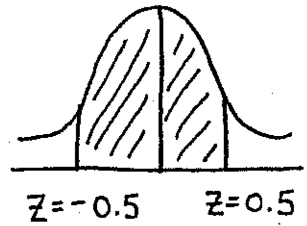
a- $P(38 < x < 40) = ?$



convert to Z-values

$$Z = \frac{38-39}{2} = -\tfrac{1}{2}$$

$$Z = \frac{40-39}{2} = \tfrac{1}{2}$$



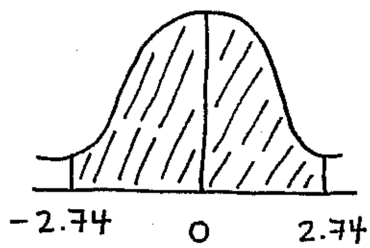
$$P(-0.5 < Z < 0.5) = 0.1915 + 0.1915 = \boxed{0.383}$$

b- $P(38 < \bar{x} < 40) = ?$

convert to Z-values

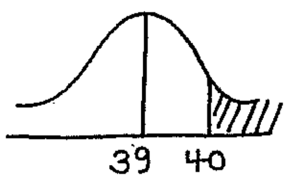
$$Z = \frac{38-39}{2/\sqrt{30}} = -2.74$$

$$Z = \frac{40-39}{2/\sqrt{30}} = 2.74$$

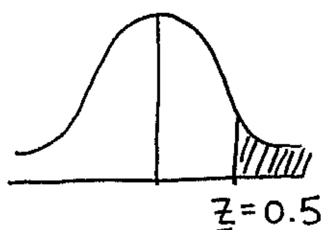


$$P(-2.74 < Z < 2.74) = 0.4969 + 0.4969 = \boxed{0.9938}$$

c- $P(x > 40)$



CONVERT TO Z-VALUES

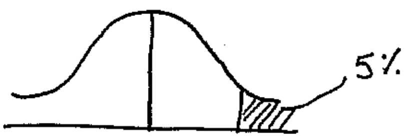


$$P(Z > 0.5) = 0.5 - 0.1915 = \boxed{0.3085}$$

d- $P(\bar{x} > 40) = P(Z > 2.74) = 0.5 - 0.4969 = \boxed{0.0031}$

2

$n = 200$
$\bar{x} = 14.3$
$\sigma = 2.5$
$\alpha = 0.10$



$Z_{\alpha/2} = Z_{0.05}$

$$E = \pm \frac{Z_{\alpha/2}\,\sigma}{\sqrt{n}}$$

$$= \pm \frac{1.64\,(2.5)}{\sqrt{200}} = \pm 0.29$$

90% C.I is $(\bar{x} - 0.29 \text{ to } \bar{x} + 0.29)$

$$\boxed{14.01 < \mu < 14.59}$$

3

$\sigma = 1$
$n = ?$
$E = 0.5$
$\alpha = 0.01$

$Z_{\alpha/2} = Z_{0.005}$
$= 2.57$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

$$= \left(\frac{(2.57)(1)}{0.5}\right)^2$$

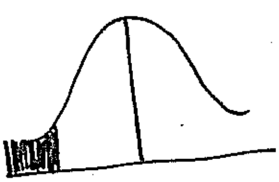
$$= 26.42$$

Round up sample size of
$\boxed{\text{27 required}}$

4

$H_0: \mu \geq 26.4$
$H_a: \mu < 26.4$

$n = 64$
$s = 1.2$
$\bar{x} = 23.6$
$\alpha = 0.05$



$t = -1.669$

$dF = 63$

$$t = \frac{23.6 - 26.4}{1.2/\sqrt{64}} = -18.67$$

We would $\boxed{\text{Reject } H_0}$

5 $\sigma = 5$
$n = 100$
$\bar{x} = 40.6$

(a) $Z_{\alpha/2} = Z_{0.025} = 1.96$

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96\left(\frac{5}{\sqrt{100}}\right) = 0.98$$

$$\bar{x} - 0.98 < \mu < \bar{x} + 0.98$$

$$\boxed{39.62 < \mu < 41.58}$$

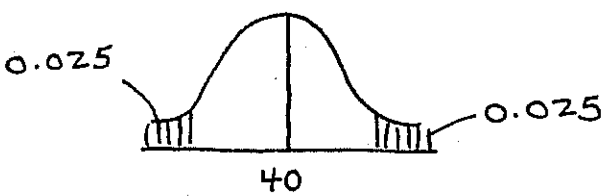
(b)

$H_0 : \mu = 40$

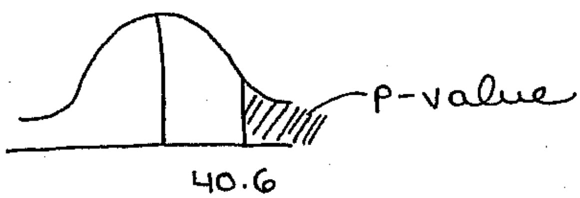
$H_a : \mu \neq 40$

Since $n = 100$ $(n \geq 30)$ we can use z-table

P-value approach (0.05 significance)



p-value is probability of being as extreme as $\bar{x} = 40.6$



$$Z = \frac{40 - 40.6}{5/\sqrt{100}} = 1.2$$

$$P(Z > 1.2) = 0.5 - 0.3849$$
$$= 0.1151$$

p-value is larger than $\alpha = 0.05$

so we <u>do NOT reject H_0</u>.

6 $\sigma = 10$
$n = 25$
$\bar{x} = 95$

PARENT POPULATION IS NORMAL
SO WE CAN USE Z-TABLE

(a) $\alpha = 0.20$

$Z_{\alpha/2} = Z_{0.10} = 1.28$

$$E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 1.28 \cdot \frac{10}{\sqrt{25}} = 2.56$$

80% C.I is $\bar{x} - 2.56 < \mu < \bar{x} + 2.56$

$$\boxed{92.44 < \mu < 97.56}$$

(b) $E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $n = 100$

$= 1.28 \left(\frac{10}{\sqrt{100}} \right) = 1.28$

80% C.I is $\bar{x} - 1.28 < \mu < \bar{x} + 1.28$

$$\boxed{93.72 < \mu < 96.28}$$

(c) $\sigma = 5$
$n = 25$

$$E = 1.28 \cdot \frac{5}{\sqrt{25}} = 1.28$$

80% C.I. is $\boxed{93.72 < \mu < 96.28}$

7

$H_0: \mu = 48$

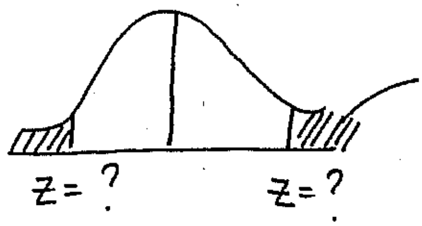
$H_a: \mu \neq 48$

p. value is 0.102

$\sigma = 3.5$

$\bar{x} = ?$

$n = 75$



$\frac{0.102}{2} = 0.051$

$z = 1.64$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow 1.64 = \frac{\bar{x} - 48}{3.5/\sqrt{75}}$$

$$\boxed{\bar{x} = 48.66}$$

OR $z = -1.64$

$$-1.64 = \frac{\bar{x} - 48}{3.5/\sqrt{75}}$$

OR $\boxed{\bar{x} = 47.33}$

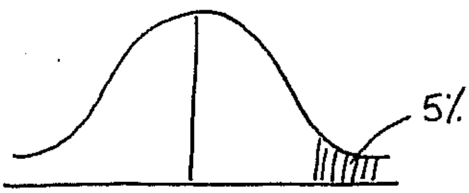
(8)

$n = 12$
$\bar{x} = 92460$
$S = 5200$
$df = 11$
$\alpha = 5\%$

- σ is UNKNOWN so we use t-test
- PARENT POPULATION is NORMAL so we CAN use t-table

$H_0: \mu \le 88950$

$H_a: \mu > 88950$



$t = 1.796$
$df = 11$

REJECTION REGION

$t_\alpha = 1.796$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

$$= \frac{92460 - 88950}{5200/\sqrt{12}} = 2.338$$

since $t > 1.796$
we $\boxed{\text{reject } H_0}$

- HOUSES ON THE OUTSKIRTS OF TOWN HAVE A HIGHER Value, AT 5% SIGNIFICANCE

(9)

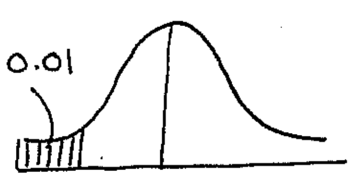
$n = 31$
$\bar{x} = 19.4$
$S = 9.6$
$\alpha = 0.01$
$t_{0.01} = -2.457$
dF 30

$n \geq 30$ so we CAN use the t-table

$H_0: \mu \geq 25$

$H_a: \mu < 25$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{19.4 - 25}{9.6/\sqrt{31}} = -3.25$$



$t = -2.457$ REJECTION

$\boxed{\text{REJECT } H_0}$
Student's CLAIM CAN BE REJECTED.

(10)

$$\bar{x} = \frac{\sum x}{n} = \frac{3952}{24} = 164.67$$

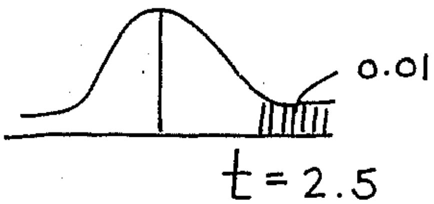
$$S = 17.559$$

calculator

since weights are normal we can use t-table

$\alpha = 0.01$

$df = 23$



$H_0: \quad \mu \leq 160$

$H_a: \quad \mu > 160$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{164.67 - 160}{17.559/\sqrt{24}}$$

$$= 1.30$$

$\boxed{\text{DO NOT REJECT } H_0}$

MEN'S weights do NOT exceed 160.

(11)

$n = 60$

$x = 22$

$p' = 22/60$

$\alpha = 0.01$

$Z_{\alpha/2} = Z_{0.005} = 2.57$

since $np' = 22$

$\&\ n(1-p') = 38$

are ≥ 5

we can use z-table

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{p'(1-p')}{n}}$$

$$= 2.57 \cdot \sqrt{\frac{(22/60)(38/60)}{60}}$$

$$= 0.1598$$

$$p' - E < p < p' + E$$

$$22/60 - 0.1598 < p < 22/60 + 0.1598$$

$$\boxed{0.207 < P < 0.527}$$

12 a- $H_0: p \le 0.6$
$H_a: p > 0.6$

e- $H_0: p \le 0.5$
$H_a: p > 0.5$

b- $H_0: p \geq 0.5$
$H_a: p < 0.5$

f- $H_0: p \geq 0.75$
$H_a: p < 0.75$

c- $H_0: p \le 1/3$
$H_a: p > 1/3$

g- $H_0: p = 0.5$
$H_a: p \neq 0.5$

d- $H_0: p \geq 0.5$
$H_a: p < 0.5$

13

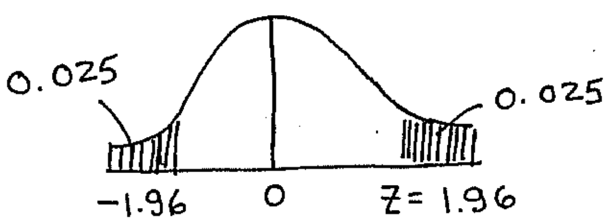
$p' = \frac{50}{100} = 0.5$

$H_0: p = 0.6$
$H_a: p \neq 0.6$

$n = 100$

$$\sigma_0 = \sqrt{\frac{(0.5)(0.5)}{100}} = 0.0595$$

since $np_0 = 100 \cdot (0.5) = 50 \geq 5$
$n(1-p_0) = 100 \cdot (0.5) = 50 \geq 5$
we can use z-table



Test Statistic

$$z = \frac{p' - p_0}{\sigma_0} = \frac{0.5 - 0.6}{0.0595} = -1.68$$

Do $\boxed{\text{Not Reject } H_0}$

The politician

(14)

$H_0: \; p \geq 0.9$

$H_a: \; p < 0.9$ (Fewer than 90% of claims)

$n = 75$

$p' = 55/75$

$\alpha = 5\%$

$\sigma_0 = \sqrt{\dfrac{(55/75)(20/75)}{75}}$

$= 0.051$

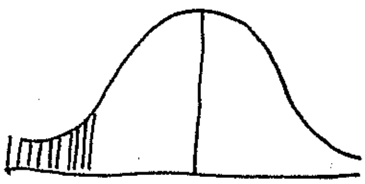
$np_0 = 75(0.9) = \quad \geq 5$

$n(1-p_0) = 75(0.1) = 7.5 \geq 5$

So we can use z-table

Calculate p-value:

$$Z = \frac{p' - p_0}{\sigma_0} = \frac{55/75 - 0.9}{\sigma_0} = -3.26$$



-3.26

probability of having a value this extreme is p-value:

$P(Z < -3.26) = 0.5 - 0.4994$

$= 0.0006$

we $\boxed{\text{REJECT } H_0}$

The insurance company is correct.

(15) $n = 250$

$p' = \dfrac{110}{250}$

$\alpha = 0.05$

Since

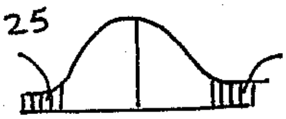
$np' = 250\left(\dfrac{110}{250}\right) = 110 \geq 5$

$n(1-p') = 140 \geq 5$

we can use z-table

$Z_{\alpha/2} = Z_{0.025} =$

0.025 0.025



$z = 1.96$

$$E = Z_{\alpha/2} \cdot S = Z_{\alpha/2} \cdot \sqrt{\frac{\left(\frac{110}{250}\right)\left(\frac{140}{250}\right)}{250}}$$

$= (1.96) \cdot (0.031)$

$= 0.0615$

$p' - E < p < p' + E$

$\boxed{-0.018 < p < 0.502}$