Quiz 2

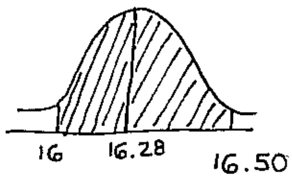
Business Statistics (201-934-DW)
Wednesday November 4th 2009
Instructor Emilie Richer

NAME SOLUTIONS

1- *(10 marks)*
In a photographic process, the develping time of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation of 0.12 second. Find the probability that it will take

a. anywhere from 16.00 to 16.50 seconds to develop one of the prints;
b. at least 16.20 seconds to develop one of the prints;
c. at most 16.35 seconds to develop one of the prints

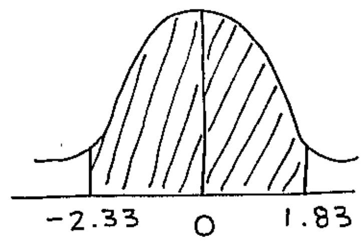
a.



convert to z-values

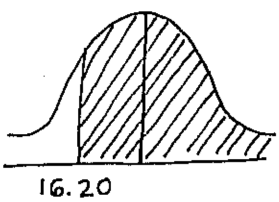
$$Z = \frac{16.5 - 16.28}{0.12} = 1.83$$

$$Z = \frac{16 - 16.28}{0.12} = -2.33$$

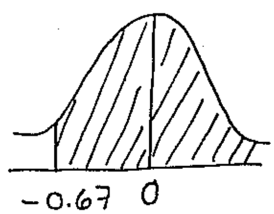


$P(-2.33 < Z < 1.83)$
$= P(16.00 < X < 16.50)$
$= 0.4664 + 0.4901$
$= \boxed{0.9565}$

b.

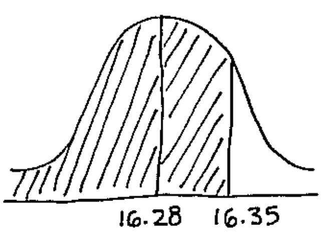


$$Z = \frac{16.2 - 16.28}{0.12}$$
$$= -0.67$$

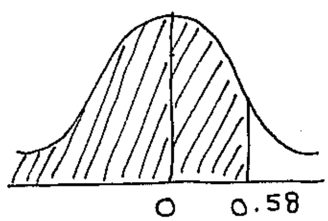


$P(Z > -0.67)$
$= P(X > 16.2)$
$= 0.5 + 0.2794 = \boxed{0.7794}$

c.



$$Z = \frac{16.35 - 16.28}{0.12}$$
$$= 0.58$$



$P(Z < 0.58)$
$= P(X < 16.35) = 0.5 + 0.2190$
$\boxed{= 0.719}$

2- *(5 marks)*
A manufacturer knows that, on average 2% of the electric toasters that he makes will require repairs within 90 days after they are sold. Use the *normal approximation to the binomial distribution* to determine the probability that among 1200 of these toasters, less than 30 will require repairs within the first 90 days after they are sold.

Let x = # of toasters needing repairs

This is binomial with $n = 1200$, $p = 0.02$, $q = 0.98$

we can find μ & σ for this distribution

$$\mu = np = 1200(0.02) = 24$$

$$\sigma = \sqrt{npq} = \sqrt{1200(0.02)(0.98)} = 4.85$$

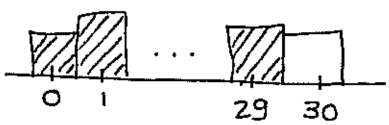
we want $P(0 \le x < 30)$

check whether we can use normal approximation

$np = 24$

$nq = 1176$ both are ≥ 5 so the approximation is valid

we want $P(0 \le x < 30)$

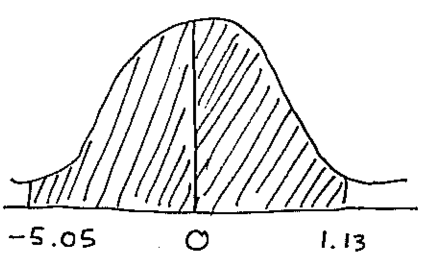


with continuity correction

$$P(-0.5 < x < 29.5)$$

convert to $z = \dfrac{-0.5 - 24}{4.85} = -5.05$

$$z = \frac{29.5 - 24}{4.85} = 1.13$$



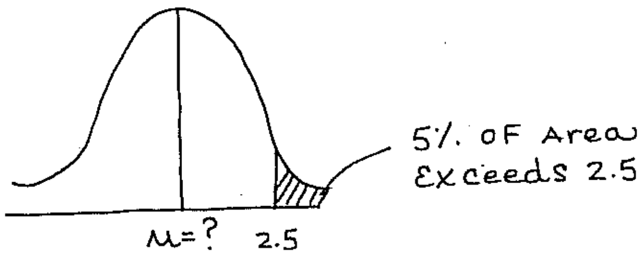
$P(\quad < z < 1.13)$

$= 0.5 + 0.3708$

$= \boxed{0.8708}$

3- *(5 marks)*

A stamping machine produces can tops whose diameters are normally distributed with a standard deviation of 0.01 inch. At what mean diameter should the machine be set so that no more than 5% of the can tops produced have diameters exceeding 2.5 inches?



Z-value corresponding To 5% is the value closest to 0.4500 in the table:

$$Z = 1.64$$

$$Z = \frac{X-\mu}{\sigma}$$

$$1.64 = \frac{2.5-\mu}{0.01}$$

$$0.0164 = 2.5-\mu$$

$$\mu = 2.5-0.0164$$

$$= 2.4836$$

The diameter should be set at $\boxed{\mu = 2.4836}$

4 - *(10 marks)*

A student comes into a test completely unprepared. Luckily for him the test is multiple choice! There are 10 questions on the test, each question has 4 choices of answer (1 correct and 3 wrong). Since the student is unprepared, he decides that he will randomly select an answer for each question and hope for the best ... Uh Oh!!!

a. Calculate the probability that the student gets a mark of 70% on the test.

b. Calcualte the probability that the student will get a mark under 90% on the test.

THIS IS A BINOMIAL EXPERIMENT WITH

x = # OF CORRECT ANSWERS

$n = 10$

$p = 1/4 = 0.25$

$q = 3/4 = 0.75$

a. $P(x=7) = \binom{10}{7}(0.25)^7(0.75)^3 = \boxed{0.00309}$

b. $P(x < 9) = P(0) + P(1) + \cdots + P(8)$

OR $= 1 - (P(9) + P(10))$

$= 1 - \left(\binom{10}{9}(0.25)^9(0.75)^1 + \binom{10}{10}(0.25)^{10}(0.75)^0\right)$

$= 1 - (0.00002861 + 0.000000954)$

$= \boxed{0.9999704}$

SOLUTIONS

Quiz 2 – Version 2

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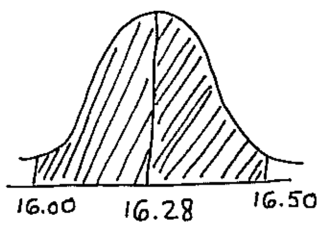
1- *(10 marks)*
In a photographic process, the develping time of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation of 0.14 second. Find the probability that it will take

a. anywhere from 16.00 to 16.50 seconds to develop one of the prints;
b. at most 16.20 seconds to develop one of the prints;
c. at least 16.35 seconds to develop one of the prints

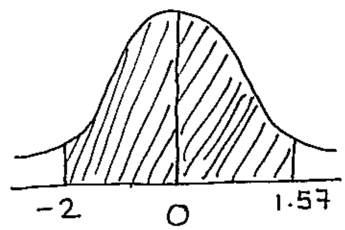
a.



convert to z-values

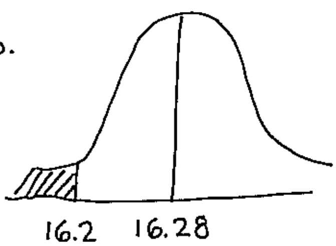
$$z = \frac{16.5 - 16.28}{0.14} = 1.57$$

$$z = \frac{16 - 16.28}{0.14} = -2$$

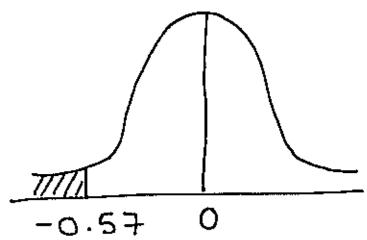


$$P(-2 < z < 1.57)$$
$$= P(16 < x < 16.5)$$
$$= 0.4772 + 0.4418$$
$$= \boxed{0.919}$$

b.

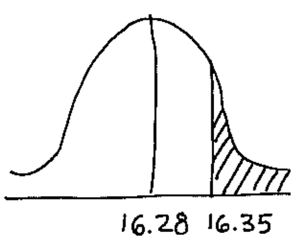


$$z = \frac{16.2 - 16.28}{0.14}$$
$$= -0.57$$

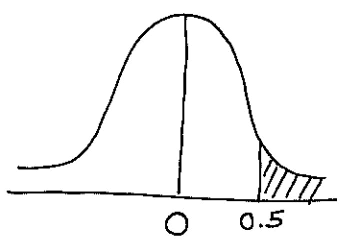


$$P(z < -0.57)$$
$$= P(x < 16.2)$$
$$= 0.5 - 0.2157$$
$$= \boxed{0.2843}$$

c.



$$z = \frac{16.35 - 16.28}{0.14}$$
$$= 0.5$$



$$P(z > 0.5)$$
$$= P(x > 16.35) = 0.5 - 0.1915$$
$$= \boxed{0.3085}$$

2- *(5 marks)*
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Let x = # of toasters needing repairs

This is binomial with $n = 1200$

$p = 0.02$

$q = 0.98$

We can find μ & σ for this distribution

$$\mu = n \cdot p = 1200(0.02) = 24$$

$$\sigma = \sqrt{npq} = \sqrt{1200(0.02)(0.98)} = 4.85$$

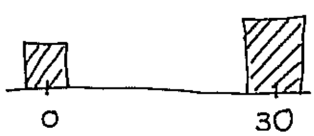
We want $P(0 \leq x \leq 30)$

Check whether we can use normal approximation

$np = 24$ $\quad nq = 1176$ $\quad$ both are ≥ 5 so we can use normal approximation.

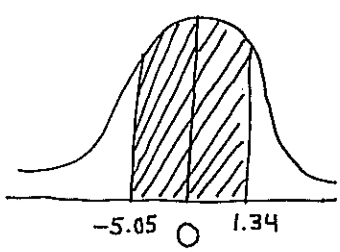
with continuity correction we want

$P(-0.5 < x < 30.5)$



convert to z-values $\quad z = \dfrac{-0.5 - 24}{4.85} = -5.05$

$$z = \frac{30.5 - 24}{4.85} = 1.34$$



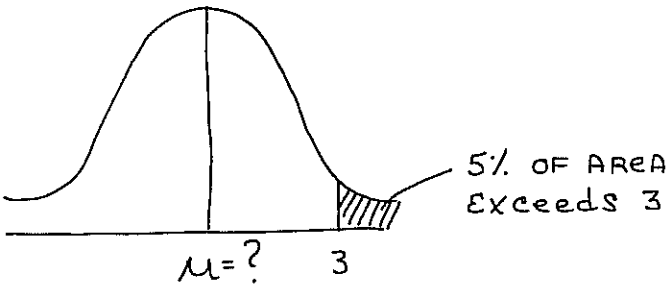
$P(-5.05 < z < 1.34)$

$= 0.5 + 0.4099$

$= \boxed{0.9099}$

VERSION 2
SOLUTIONS

3- *(5 marks)*
A stamping machine produces can tops whose diameters are normally distributed with a standard deviation of 0.01 inch. At what mean diameter should the machine be set so that no more than 5% of the can tops produced have diameters exceeding 3 inches?



Z-value corresponding to 5% is the value closest to 0.4500 in the table:

$Z = 1.64$

$$Z = \frac{x - \mu}{\sigma}$$

$$1.64 = \frac{3 - \mu}{0.01}$$

$$0.0164 = 3 - \mu$$

$$\mu = 3 - 0.0164$$

$$= 2.9836$$

The diameter should be set at $\boxed{\mu = 2.9836}$

4 - *(10 marks)*
A student comes into a test completely unprepared. Luckily for him the test is multiple choice! There are 10 questions on the test, each question has 4 choices of answer (1 correct and 3 wrong). Since the student is unprepared, he decides that he will randomly select an answer for each question and hope for the best... Uh Oh!!!

a. Calculate the probability that the student gets a mark of 60% on the test.
b. Calcualte the probability that the student gets a mark over 20% on the test.

THIS IS A BINOMIAL EXPERIMENT WITH

x = # OF CORRECT ANSWERS

$n = 10$

$p = 1/4 = 0.25$

$q = 3/4 = 0.75$

(a) $P(x=6) = \binom{10}{6}(0.25)^6(0.75)^4 = \boxed{0.0162}$

(b) $P(x>2) = P(3) + P(4) + \cdots + P(10)$

OR $= 1 - \left(P(0) + P(1) + P(2)\right)$

$$= 1 - \left(\binom{10}{0}(0.25)^0(0.75)^{10} + \binom{10}{1}(0.25)^1(0.75)^9 + \binom{10}{2}(0.25)^2(0.75)^8\right)$$

$$= 1 - \left(0.0563 + 0.1877 + 0.28157\right)$$

$$= 1 - 0.526$$

$$\boxed{= 0.47}$$