

VERSION 1 SOLUTIONS

Question 1 (8 marks)

One thousand employees at the Consume-O-Tron Company were polled about worker satisfaction. The sex of each employee was recorded as well as their skill level. The following results were recorded:

| | Male | | Female | | Total |
|-------------|---------|-----------|---------|-----------|-------|
| | Skilled | Unskilled | Skilled | Unskilled | |
| Satisfied | 350 | 150 | 25 | 100 | 625 |
| Unsatisfied | 150 | 100 | 75 | 50 | 375 |
| | 500 | 250 | 100 | 150 | 1000 |

One of the thousand employees polled is selected at random.

(a) (1 marks) Find the probability that the worker is unskilled.

$$\frac{250 + 150}{1000} = \frac{400}{1000} = \boxed{0.4}$$

(b) (1 marks) Find the probability that the person is an unskilled, male worker.

$$\frac{250}{1000} = \boxed{0.25}$$

(c) (2 marks) If the person is female, what is the probability that they are satisfied with their work?

$$\frac{125}{250} = \boxed{0.5}$$

(d) (2 marks) If the person is an unskilled, male worker, what is the probability that they are unsatisfied with their job?

$$\frac{100}{250} = \boxed{0.4}$$

(e) (2 marks) Is skill level independent from job satisfaction?

Let S = SATISFIED
 U = UNSKILLED

We check if $P(S|U) = P(S)$

$$P(S) = \frac{625}{1000} = 0.625$$

$$P(S|U) = \frac{250}{400} = 0.625$$

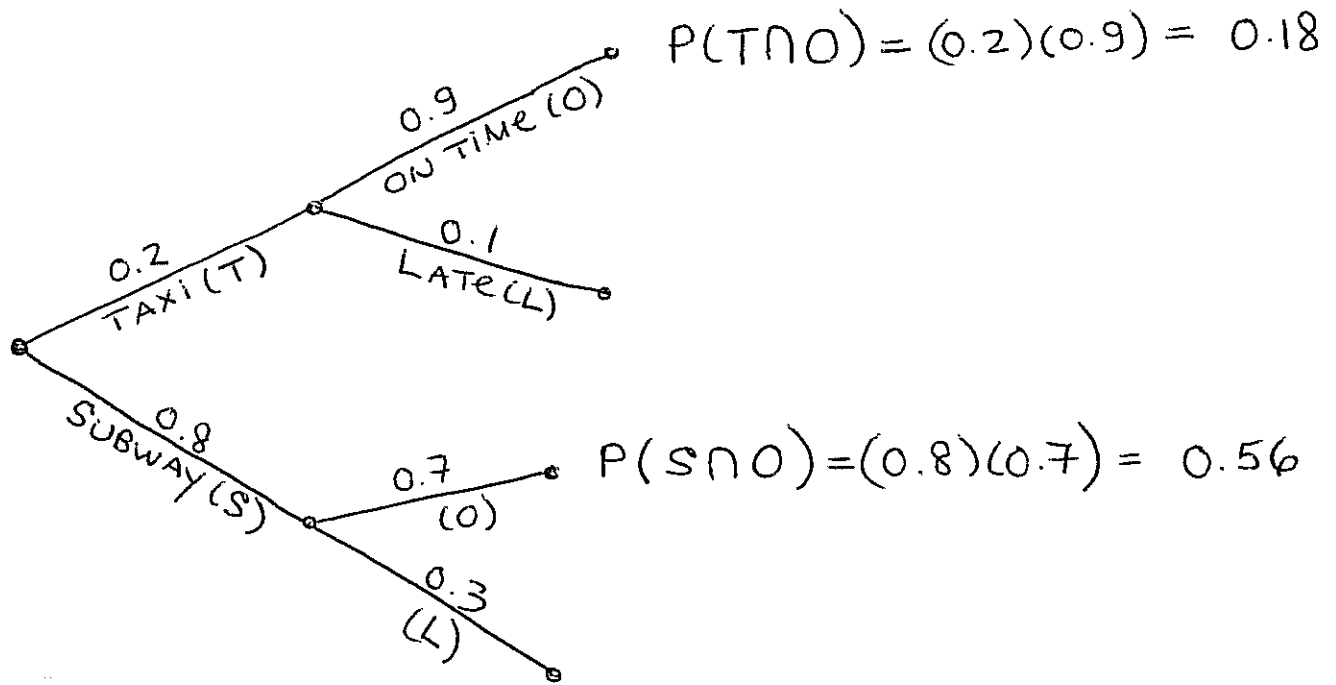
Yes THEY ARE INDEPENDENT

Question 2 (8 marks)

Gino lives in a large city and commutes to school daily by subway or by taxi. He takes the subway 80% of the time because it costs less, and he takes a taxi the other 20% of the time. When taking the subway he arrives at school on time 70% of the time, whereas he makes it on time 90% of the time when traveling by taxi.

(a) What is the probability that Gino arrives on time on any given day?

(b) If Gino arrived on time to school on a given day, what is the probability that he took a taxi to get there?



(a) $P(O) = 0.18 + 0.56 = \boxed{0.74}$

(b) $P(T|O) = \frac{P(T \cap O)}{P(O)} = \frac{0.18}{0.74} = \boxed{0.243}$

Question 3 (4 marks)

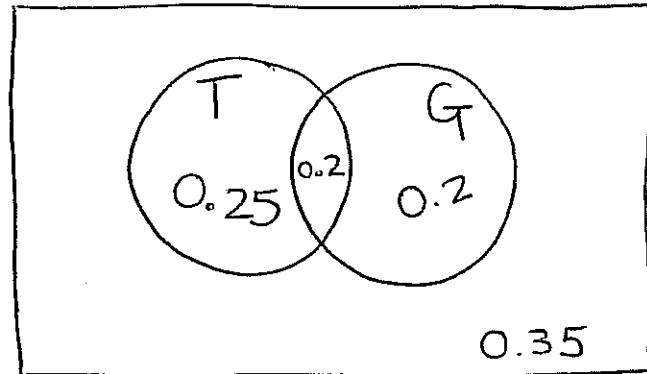
A city has two main newspapers, the *Times* and the *Gazette*. Readership of the papers among city residents is distributed as follows:

65% of people read at least one of the ~~two~~ newspapers every day

45% read the Times

40% read the Gazette

(a) Draw a Venn diagram representing the readership of the two papers (using the letters T, G)



(b) What is the probability that a resident reads both the Times and the Gazette?

$$P(T \cup G) = 0.65$$

$$P(T) = 0.45$$

$$P(G) = 0.4$$

$$\begin{aligned} P(T \cap G) &= P(T) + P(G) - P(T \cup G) \\ &= 0.45 + 0.4 - 0.65 \\ &= 0.2 \end{aligned}$$

(c) What is the probability that a resident reads *only* the Times?

As we can see from the Venn diagram
0.25 is the prob that only the Times
is read

Question 4 (10 marks)

Midterm grades of 30 students in a Finance class are recorded below:

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 61 | 21 | 85 | 62 | 97 | 71 | 55 | 58 | 10 | 86 |
| 71 | 77 | 57 | 68 | 78 | 82 | 84 | 90 | 70 | 59 |
| 62 | 59 | 84 | 90 | 74 | 82 | 71 | 65 | 93 | 70 |

(a) (4 marks) Find the sample mean and the sample standard deviation.

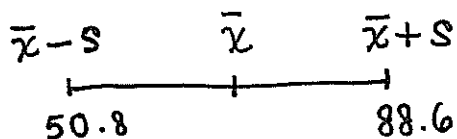
Using calculator

$$\bar{x} = 69.7 \quad S = 18.9$$

(b) (4 marks) Does the data follow the empirical rule?

SORTED DATA

| | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 10 | 21 | 55 | 57 | 58 | 59 | 59 | 61 | 62 | 62 | 65 | 68 | 70 | 70 | 71 |
| 71 | 71 | 74 | 77 | 78 | 82 | 82 | 84 | 84 | 85 | 86 | 90 | 90 | 93 | 97 |



There are $\frac{24}{30} = 80\%$ of data within 1S of THE MEAN

Thus the data does NOT follow the empirical rule which states that approx. 68% should be in that interval

(c) (2 marks) What is the z-score of a student with a grade of 65? Explain what this means using the statistical vocabulary mean and standard deviation.

$$Z = \frac{65 - 69.7}{18.9} = -0.25$$

This means they are ABOUT $\frac{1}{4}$ of a STANDARD deviation below the mean.

Question 5 (6 marks)

Given $P(R) = 0.5$ and $P(S) = 0.3$, and given that the events R and S are independent, find the following.

(a) $P(R \cap S)$

$$P(R|S) = P(R) \text{ independent}$$

$$\text{so } P(R \cap S) = (0.5)(0.3) = \boxed{0.15}$$

(b) $P(R \cup S)$

$$= P(R) + P(S) - P(R \cap S)$$

$$= 0.5 + 0.3 - 0.15 = \boxed{0.65}$$

(c) $P(S')$

$$= 1 - P(S)$$

$$= 1 - 0.3 = \boxed{0.7}$$

(d) $P(R|S)$

Because they are independent

$$P(R|S) = P(R) = \boxed{0.5}$$

(e) $P(S'|R)$

Similarly

$$P(S'|R) = P(S') = \boxed{0.7}$$

(f) Are events R and S mutually exclusive? Explain.

NO BECAUSE THEIR INTERSECTION IS NOT empty $P(R \cap S) = 0.15$

Question 6 (10 marks)

The speeds (in km/h) of 26 cars were recorded by a radar device at a busy city intersection. The results are listed below

35 62 55 61 35 47 42 66 48 42 51 53 60
47 42 50 51 54 62 53 44 41 41 35 52 57

(a) Find the 30th and 80th percentiles (P_{30} and P_{80}) of the above data. (Note: If your calculator gives percentiles, you must still show all the work that would be required if this was not the case)

35 35 35 41 41 42 42 42 44 47 47 48 50 51 51 52 53
53 54 55 57 60 61 62 62 66

$(0.3)26 = 7.8 \rightarrow$ Round up to 8
 $(0.8)26 = 20.8 \rightarrow$ Round up to 21

| |
|---------------|
| $P_{30} = 42$ |
| $P_{80} = 57$ |

(b) Find the sample mode

| |
|---------|
| NO Mode |
|---------|

(c) Find the sample median

$$\frac{50 + 51}{2} = 50.5$$

(d) Find the sample range

$$66 - 35 = 31$$

(e) Find the sample mean

| |
|------------------|
| $\bar{x} = 49.5$ |
|------------------|

(f) Find the sample standard deviation

| |
|------------|
| $s = 8.85$ |
|------------|

(g) Find $SS(X)$, the total variation in X

$$SS(X) = \sum x^2 - \frac{(\sum x)^2}{n} = 65566 - \frac{(1286)^2}{26} = 1958.46$$

(h) According to the sample, would you predict that the average speed at this intersection is higher than the speed limit there which is 50km/h?

no because \bar{x} is less than 50 km/hr

Question 7 (4 marks)

(a) Briefly define nCr and nPr .

nPr is the number of orderings of r objects picked from a group of n objects

nCr is the number of ways of picking r objects from a group of n objects where the order in which objects are picked does NOT MATTER

(b) How many different ways can you line up 5 people chosen from a group of 12 people?

$12P_5$ ways

(c) How many ways can you pick a 3 person committee from a group of 15 people?

$15C_3$ ways

BONUS (2.5 marks)

Two flower seeds are randomly selected from a package that contains five seeds for red flowers and three seeds for white flowers.

(a) What is the probability that both seeds will result in red flowers?

(b) What is the probability that one of each color is selected?

(c) What is the probability that both seeds will result in white flowers?

SAMPLE SPACE : # OF WAYS OF PICKING 2 FROM 8
That is $8C_2 = 28$

(a) 2 Red FROM 5 possible red seeds

$$5C_2 : \frac{10}{28} = \boxed{0.36}$$

(b)
$$\frac{5C_1 3C_1}{28} = \frac{15}{28} = \boxed{0.54}$$

(c)
$$\frac{3C_2}{28} = \frac{3}{28} = \boxed{0.11}$$