

TEST 1 - Business Statistics

201-934-DW 02

Friday October 2nd 2009

Instructor: Emilie Richer

Name: SOLUTIONS/VERSION 2

INSTRUCTIONS

- The test is marked out of 50 marks
- You have 75 minutes to complete the test
- Any calculator is permitted

SHOW ALL YOUR WORK

This means:

- Define any variables that you are using
- Indicate which formulas you are using
- Don't simply put an answer down (unless it comes directly from your calculator)

**ANY TALKING AFTER THE TEST HAS BEGUN WILL RESULT IN A
MARK OF ZERO**

FORMULAS

$$\bar{x} = \frac{\sum X}{n}$$

$$s^2 = \frac{SS(X)}{n-1}$$

$$SS(X) = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$z = \frac{x - \bar{x}}{s}$$

$$s = \sqrt{s^2}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cap B) = P(A|B) P(B)$$

$$P(A \cap B) = P(A) P(B) \text{ *If A and B are independent}$$

$$P(A') = 1 - P(A)$$

Question 1 (8 marks)

One thousand employees at the Consume-O-Tron Company were polled about worker satisfaction. The sex of each employee was recorded as well as their skill level. The following results were recorded:

	Female		Male		Total
	Unskilled	Skilled	Unskilled	Skilled	
Satisfied	350	150	25	100	625
Unsatisfied	150	100	75	50	375
	500	250	100	150	1000

One of the thousand employees polled is selected at random.

- (a) (1 marks) Find the probability that the worker is unskilled.

$$\frac{500 + 100}{1000} = \boxed{0.6}$$

- (b) (1 marks) Find the probability that the person is an unskilled, male worker.

$$\frac{100}{1000} = \boxed{0.1}$$

- (c) (2 marks) If the person is female, what is the probability that they are satisfied with their work?

$$\frac{350 + 150}{750} = \frac{500}{750} = 66.67\% = \boxed{0.667}$$

- (d) (2 marks) If the person is an unskilled, male worker, what is the probability that they are unsatisfied with their job?

$$\frac{75}{100} = \boxed{0.75}$$

- (e) (2 marks) Is skill level independent from job satisfaction?

Let $S = \text{skilled}$
 $UN = \text{Unsatisfied}$
 We check that
 $P(UN) = P(UN|S)$

$$P(UN) = \frac{375}{1000} = 0.375$$

$$P(UN|S) = \frac{100 + 50}{250 + 150} = 0.375$$

so $\boxed{\text{YES}}$ they are independent

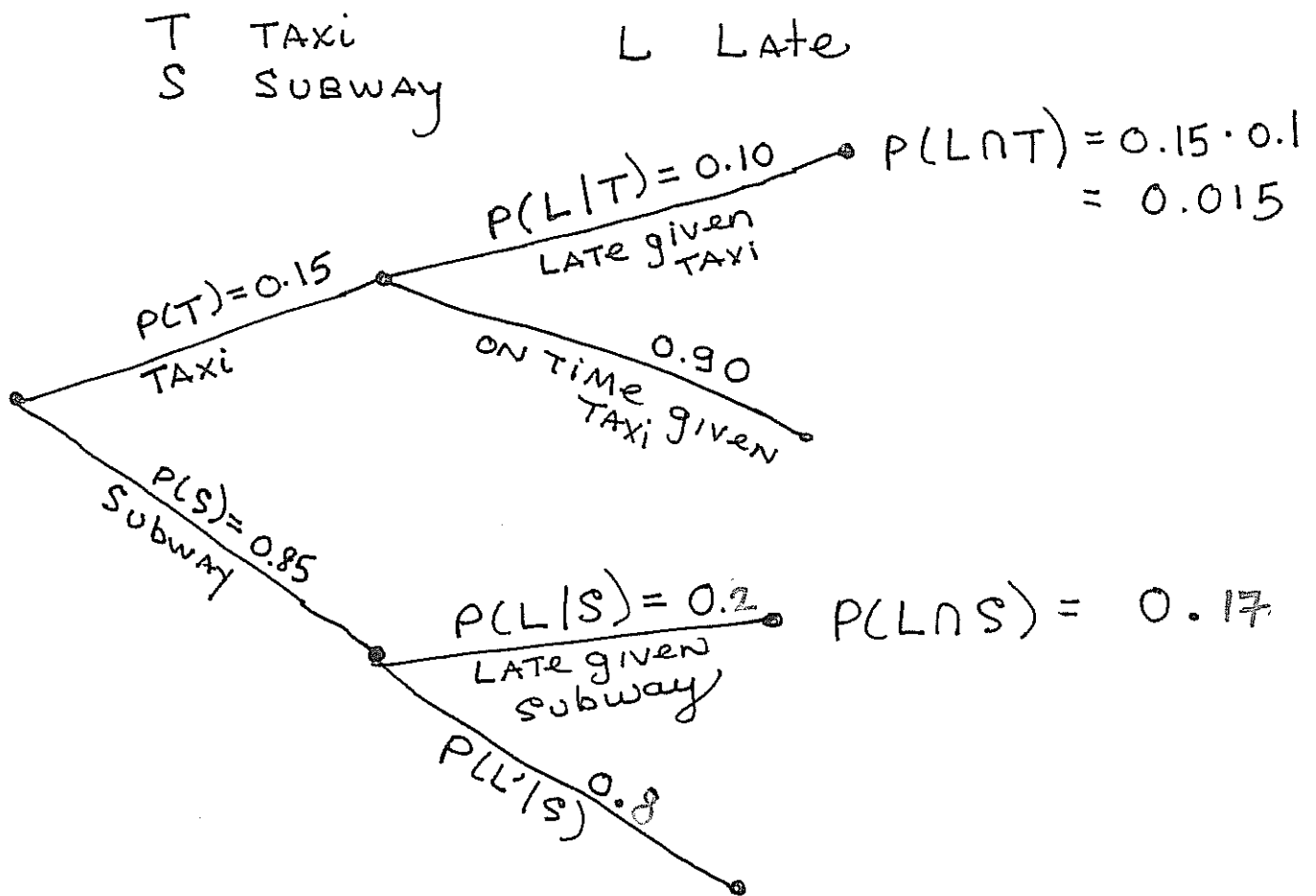
Question 2 (8 marks)

Gino lives in a large city and commutes to school daily by subway or by taxi. He takes the subway 85% of the time because it costs less, and he takes a taxi the other 15% of the time.

When taking the subway he arrives at school on time 80% of the time, whereas he makes it on time 90% of the time when traveling by taxi.

(a) What is the probability that Gino arrives late to school on any given day?

(b) If Gino arrived late to school on a given day, what is the probability that he took a taxi to get there?



$$\begin{aligned} (a) \quad P(L) &= P(LNT) + P(LNS) \\ &= 0.015 + 0.17 \\ &= \boxed{0.185} \end{aligned}$$

$$(b) \quad P(T|L) = \frac{P(LNT)}{P(L)} = \frac{0.015}{0.185} = \boxed{0.081}$$

Question 3 (4 marks)

A city has two main newspapers, the *Times* and the *Gazette*. Readership of the papers among city residents is distributed as follows:

75% of people read at least one of the two newspapers every day

55% read the Times

40% read the Gazette

(a) What is the probability that a resident reads both the Times and the Gazette?

$$P(T \cup G) = 0.75$$

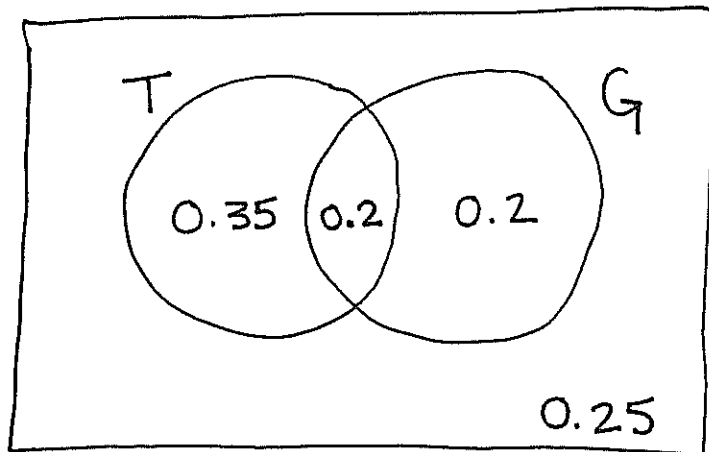
$$P(T) = 0.55$$

$$P(G) = 0.40$$

$$P(T \cup G) = P(T) + P(G) - P(T \cap G)$$
$$0.75 = 0.55 + 0.40 - P(T \cap G)$$

$$\boxed{P(T \cap G) = 0.2}$$

(b) Draw a Venn diagram representing the readership of the two papers (using the letters T, G)



(c) What is the probability that a resident reads *only* the Gazette?

$$\boxed{0.20} \quad (\text{see Venn diagram})$$

Question 4 (10 marks)

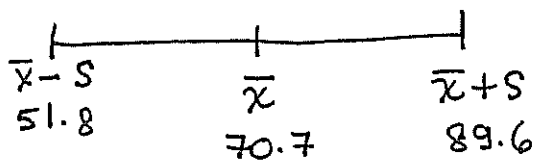
Midterm grades of 30 students in a Finance class are recorded below:

62	22	86	63	98	72	56	59	11	87
72	78	58	69	79	83	85	91	71	60
63	60	85	91	75	83	72	66	94	71

(a) (4 marks) Find the sample mean and the sample standard deviation.

$$\bar{x} = 70.7 \quad S = 18.9$$

(b) (4 marks) Does the data follow the empirical rule?



There ARE $\frac{24}{30}$ data within 1 standard deviation of \bar{x} ; this is 80% whereas in order to SATISFY the empirical rule there should be 68%.

NO

(c) (2 marks) What is the z-score of a student with a grade of 72? Explain what this means using the statistical vocabulary mean and standard deviation.

$$z = \frac{72 - \bar{x}}{S} = \frac{72 - 70.7}{18.9} = 0.068$$

This means 72 is 0.068 of 1 standard deviation ABOVE the mean.

Question 5 (6 marks)

Given $P(R) = 0.6$ and $P(S) = 0.2$, and given that the events R and S are independent, find the following.

(a) $P(R \cap S)$

$$\begin{aligned} &= P(R) \cdot P(S) && \text{(Because } R, S \text{ are independent)} \\ &= (0.6) \cdot (0.2) \\ &= \boxed{0.12} \end{aligned}$$

(b) $P(R \cup S)$

$$\begin{aligned} &= P(R) + P(S) - P(R \cap S) \\ &= 0.6 + 0.2 - 0.12 \\ &= \boxed{0.68} \end{aligned}$$

(c) $P(S')$

$$\begin{aligned} &= 1 - P(S) \\ &= \boxed{0.8} \end{aligned}$$

(d) $P(R|S)$

$$\begin{aligned} &= P(R) \\ &= \boxed{0.6} && \text{(Because } R, S \text{ independent)} \end{aligned}$$

(e) $P(S'|R)$

$$\begin{aligned} &= P(S') \\ &= \boxed{0.8} && \text{(Again due to independence)} \end{aligned}$$

(f) Are events R and S mutually exclusive? Explain.

No because $P(R \cap S) \neq 0$, that is, there is something in their intersection

Question 6 (10 marks)

The speeds (in km/h) of 26 cars were recorded by a radar device at a busy city intersection. The results are listed below

30 57 50 56 30 42 37 61 43 37 46 48 55
42 37 45 46 49 57 48 39 36 36 30 47 52

(a) Find the 20th and 70th percentiles (P_{20} and P_{70}) of the above data. (Note: If your calculator gives percentiles, you must still show all the work that would be required if this was not the case)

Sorted data

30 30 30 36 36 37 37 37 39 42 42 43 45
46 46 47 48 48 49 50 52 55 56 57 57 61

$$(0.2)26 = 5.2 \rightarrow \text{ROUND up } 6^{\text{th}}$$

$$(0.7)26 = 18.2 \rightarrow " " 19^{\text{th}}$$

$P_{20} = 37$
$P_{70} = 49$

(b) Find the sample mode

NONE

(c) Find the sample median

$$\frac{45 + 46}{2} = \boxed{45.5}$$

(d) Find the sample range

$$61 - 30 = \boxed{31}$$

(e) Find the sample mean

$$\boxed{44.5}$$

(f) Find the sample standard deviation

$$\boxed{8.85}$$

(g) Find $SS(X)$, the total variation in X

$$SS(X) = \sum x^2 - \frac{(\sum x)^2}{n} = 53356 - \frac{(1156)^2}{26} = 1958.46$$

(h) According to the sample, would you predict that the average speed at this intersection is higher than the speed limit there which is 45km/h?

$$\boxed{\text{No}} \text{ as } \bar{x} = 44.5$$

Question 7 (4 marks)

(a) Briefly define nCr and nPr .

nCr : Number of ways of picking r objects from a group of n
Order of picks does not matter

nPr : Number of Arrangements of r objects from a group of n objects

(b) In how many ways can you pick a 4 person committee from a group of 12 people?

$$12C_4$$

(c) In how many different ways can you line up 6 people chosen from a group of 15 people?

$$15P_6$$

BONUS (2.5 marks)

Two flower seeds are randomly selected from a package that contains five seeds for white flowers and three seeds for red flowers.

- (a) What is the probability that both seeds will result in red flowers?
- (b) What is the probability that one of each color is selected?
- (c) What is the probability that both seeds will result in white flowers?

TOTAL # OF WAYS OF PICKING
2 seeds $8C_2 = 28$

$$(a) \frac{3C_2}{8C_2} = \frac{3}{28} = \boxed{0.11}$$

$$(b) \frac{3C_1 5C_1}{8C_2} = \frac{15}{28} = \boxed{0.54}$$

$$(c) \frac{5C_2}{8C_2} = \frac{10}{28} = \boxed{0.36}$$