TEST 2

Business Statistics (201-934-DW)
Wednesday November 10th 2009
. Instructor Emilie Richer

SOLUTIONS

1- *(5 marks)*
A survey asked a group of medical doctors how many children they had fathered. The results are summarized by the ungrouped frequency table below:

Number of Children	0	1	2	3	4	5
Number of Doctors	15	12	26	14	4	2

a. Find the mean
b. Find the standard deviation

x	f	$x \cdot f$	$x^2 f$
0	15	0	0
1	12	12	12
2	26	52	104
3	14	42	126
4	4	16	64
5	2	10	50
	73	132	356

a. $\bar{x} = \dfrac{132}{73} = \boxed{1.808}$

b. $SS(x) = 356 - \dfrac{132^2}{73}$

$= 117.315$

$s = \sqrt{\dfrac{SS(x)}{n-1}}$

$= \sqrt{\dfrac{117.315}{72}}$

$\boxed{s = 1.276}$

2- *(8 marks)*
According to the Employee Benefit Research Institute, 48% of Canadian workers think that they will still be working when they are 65 or older. Suppose this result is true for the current population of all Canadian workers. What is the probability that in a random sample of 200 Canadian workers, less than between 75 and 90 (inclusively) will say that they will still be working when they are 65 or older. Use the (*Normal Approximation to the Binomial Distribution*).

$p = 0.48$
$q = 0.52$
$n = 200$

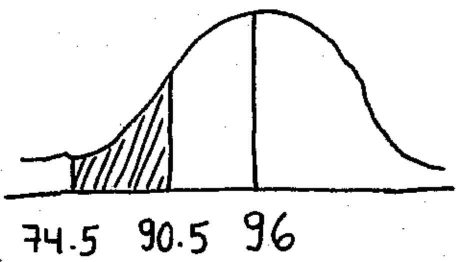
$np = 96 \geq 5$
$nq = 104 \geq 5$

we can use the approximation

$\mu = np = 96$
$\sigma = \sqrt{npq}$
$= 7.065$

we want
$P(75 \leq x \leq 90)$

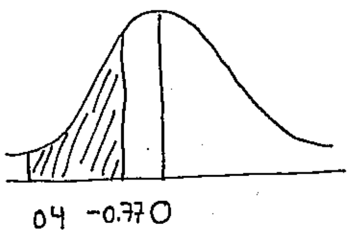
with continuity correction
$P(74.5 \leq x \leq 90.5)$



convert to z-values

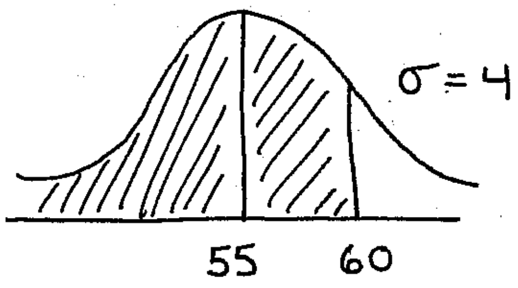
$$z = \frac{74.5 - 96}{7.065} = -3.04$$

$$z = \frac{90.5 - 96}{7.065} = -0.77$$



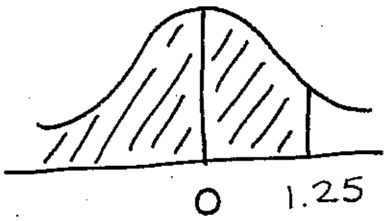
so $P(74.5 \leq x \leq 90.5)$
$= 0.4988 - 0.2794$
$\boxed{= 0.2194}$

3- *(5 marks)*
Toy racing cars are manufactured at a certain factory. The assembly times for this toy follow a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5p.m. and all assembly lines are shut at this time. If a worker starts assembling a toy racing car at 4p.m. what is the probability that he will finish the job before the company closes for the day?



One Hour To build:
we WANT
$P(X \le 60)$

$$Z = \frac{60-55}{4} = 1.25$$



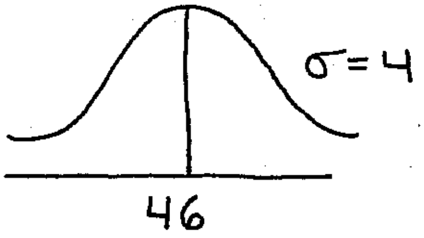
$$P(X \le 60) = 0.5 + 0.3944$$
$$= \boxed{0.8944}$$

4- *(6 marks)*

A construction zone on a highway has a posted speed limit of 40 km/h. The speeds of vehicles passing through this construction zone are normally distributed with a mean of 46 km/h and a standard deviation of 4 km/h. Find the percentage of vehicles passing through this construction zone that are

a. Exceeding the posted speed limit

b. Traveling at speeds between 50 and 55 km/h



a. $P(x > 40)$

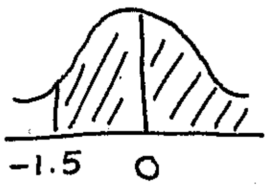
b. $P(50 < x < 55)$

convert to z-values

a. $z = \dfrac{40-46}{4} = -1.5$

$P(z > -1.5) = 0.5 + 0.4332$

$= \boxed{0.9332}$



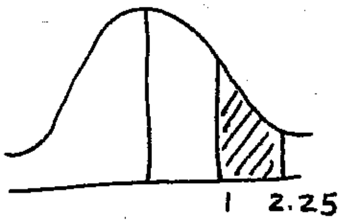
b. $z = \dfrac{50-46}{4} = 1$

$z = \dfrac{55-46}{4} = 2.25$

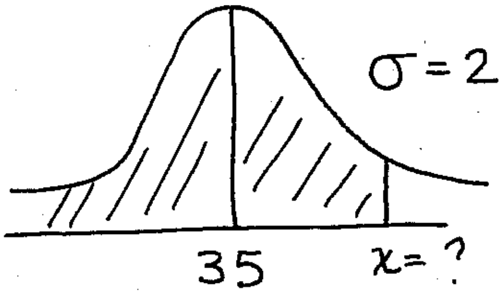
$P(1 < z < 2.25) =$

$0.4878 - 0.3413$

$= \boxed{0.1465}$



5- *(5 marks)*
Jerry knows that the time it takes him to commute to school is normally distributed with a mean of 35 minutes and a standard deviation of 2 minutes. What time must he leave home in the morning so that he is 95% sure of arriving at school by 9.a.m?



95%
OF DATA corresponds TO $z = 1.64$

$$1.64 = \frac{x - \mu}{\sigma}$$

$$1.64 = \frac{x - 35}{2}$$

$$x = 38.28 \text{ MINUTES}$$

So Jerry SHOUld leAve at roughLy

$\boxed{8:21 \text{ A.M.}}$

6- *(5 marks)*

Of the parts produced by a particular machine 0.5% are defective. If a random sample of 10 parts produced by this machine contains 2 or more defective parts, the machine shuts down for repairs. Find the probability that the machine will be shut down for repairs based on this sampling plan.

$p = 0.005$

$q = 0.995$

$n = 10$ 2 OR MORE = SHUT down

$$P(x \geq 2)$$

$$= 1 - (P(0) + P(1))$$

$$= 1 - \left({}_{10}C_0 (0.005)^0 (0.995)^{10} + {}_{10}C_1 (0.005)^1 (0.995)^9 \right)$$

$$= 1 - (0.9511 + 0.04779)$$

$$= \boxed{0.0011}$$

7- *(8 marks)*

Four men between the age of 20-30 years were tested for arm circumference and level of testosterone in their blood. The following data was collected:

	x (circumference of bicep in m)	y (level of testosterone in nanograms per hundredth of litre)	$x \cdot y$	x^2	y^2
	0.33	50	16.5	0.1089	2500
	0.29	65	18.85	0.0841	4225
	0.42	35	14.7	0.1764	1225
	0.49	32	15.68	0.2401	1024
Σ	1.53	182	65.73	0.6095	8974

a. Find the line of best fit for the above data

b. Find the coefficient of linear determination. What does this tell you about the linear relationship between bicep circumference and testosterone level?

c. Estimate the testosterone level of someone with a bicep circumference of 0.25m.

(a.)

$$SS(X) = 0.6095 - \frac{(1.53)^2}{4} = 0.024275$$

$$SS(Y) = 8974 - \frac{(182)^2}{4} = 693$$

$$SS(XY) = 65.73 - \frac{(1.53)(182)}{4} = -3.885$$

$$b_1 = \frac{SS(XY)}{SS(X)} = \frac{-3.885}{0.024275} = -160.0411$$ * don't round off yet

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$= \left(\frac{182}{4}\right) - \left(\frac{-3.885}{0.024275}\right)\left(\frac{1.53}{4}\right) = 106.716$$

$$\hat{y} = -160.04x + 106.72$$

(b.)

$$r = \frac{SS(xy)}{\sqrt{SS(x)SS(y)}} = \frac{(-3.885)}{\sqrt{(0.024275)(693)}} = -0.9472$$

This indicates weak negative linear correlation. As bicep circumference increases, testosterone levels decrease

* Note this is totally made up data

(c.)

$$\hat{y} = -160.04(0.25) + 106.72 = \boxed{66.71} \text{ ng/100th of litre}$$

8- *(8 marks)*

DATA SET:

1 2 1 7 11 8 5 7 7 8 8 11

a. Construct an ungrouped frequency table for the data above and compute the true mean and true standard deviation of the data.
b. Construct a grouped frequency table for the same data then approximate the mean and standard deviation using the class marks.

a.

x	f	$x \cdot f$	$x^2 f$
1	2	2	2
2	1	2	4
5	1	5	25
7	3	21	147
8	3	24	192
11	2	22	242
	12	76	612

$\bar{x} = 76/12 = \boxed{6.33}$

$s = \sqrt{\dfrac{612 - \frac{76^2}{12}}{11}} = \boxed{3.447}$

b. # classes $\approx \sqrt{12} = 3.46$ round down to 3

class width: $\dfrac{\text{range}}{\text{\# classes}} = \dfrac{11-1}{3} = 3.33$ round up to 4

CLASS LIMITS	f	CLASS bdries	M	$M \cdot f$	$M^2 f$
1 – 4	3	0.5 – 4.5	2.5	7.5	18.75
5 – 8	7	4.5 – 8.5	6.5	45.5	295.75
9 – 12	2	8.5 – 12.5	10.5	21	220.50
	12			74	535

$\bar{x} \approx \dfrac{74}{12} = \boxed{6.17}$

$s \approx \sqrt{\dfrac{535 - \frac{74^2}{12}}{11}} = \boxed{2.674}$