Name: Solutions

Student ID:

Test 2

Question 1. Simplify, expressing your final answer with positive exponents only:

(a) *(3 marks)*

$$\frac{a^4 b^{-5} c^{-1} a^{-1}}{b^{-2} a^3 c^7} = \frac{a^4 \cdot b^2}{b^5 c^1 a^1 a^3 c^7} = \frac{a^4 b^2}{a^4 b^5 c^8}$$

$$= \frac{1}{b^3 c^8}$$

(b) *(3 marks)*

$$(-x^{-2} y^4 z^{-5})^{44} = (-1)^{44} (x^{-2})^{44} (y^4)^{44} (z^{-5})^{44}$$

$$= (1) x^{-88} y^{176} z^{-220}$$

$$= \frac{y^{176}}{x^{88} z^{220}}$$

(c) *(4 marks)*

$$\frac{(a^2b^{-1})^{-4}ab^3}{(-2a)^3b} = \frac{a^{-8}b^4ab^3}{(-2)^3a^3b} = \frac{a^{-8}b^4ab^3}{-8a^3b}$$

$$= -\frac{ab^7}{8a^3ba^8} = -\frac{ab^6}{8a^{11}}$$

$$= -\frac{b^6}{8a^{10}}$$

(d) *(4 marks)*

$$\left(\frac{2a^{-2}b^2}{a^4b^{-3}}\right)^{-2} = \frac{2^{-2}a^4b^{-4}}{a^{-8}b^6} = \frac{a^4a^8}{2^2b^4b^6}$$

$$= \frac{a^{12}}{4b^{10}}$$

(e) *(3 marks)*

$$\frac{(14x^2y^{-3}z^{-1})^0}{(-3a)^{-2}(b^3)^2} = \frac{1}{(-3)^{-2}a^{-2}b^6} = \frac{(-3)^2a^2}{b^6}$$

$$= \frac{9a^2}{b^6}$$

Question 2. *(3 marks)* Subtract and simplify:

$$(13x^6 - 12x^5 - 3x^4 + 2x^2 - x + 5) - (-x^6 + 2x^5 - 3x^3 + 2x^2 + x - 1)$$

$$= 13x^6 - 12x^5 - 3x^4 + 2x^2 - x + 5 + x^6 - 2x^5 + 3x^3 - 2x^2 - x + 1$$

$$= 14x^6 - 14x^5 - 3x^4 + 3x^3 - 2x + 6$$

Question 3. Multiply and simplify:
(a) *(4 marks)*

$$(3x+2)^3 = (3x+2)(3x+2)(3x+2)$$

$$= (9x^2 + 6x + 6x + 4)(3x+2)$$

$$= (9x^2 + 12x + 4)(3x+2)$$

$$= 9x^2(3x+2) + 12x(3x+2) + 4(3x+2)$$

$$= 27x^3 + 18x^2 + 36x^2 + 24x + 12x + 8$$

$$= 27x^3 + 54x^2 + 36x + 8$$

(b) *(3 marks))*

$$(2x-3)(12x^2 - 2x - 5) = 2x(12x^2 - 2x - 5) - 3(12x^2 - 2x - 5)$$

$$= 24x^3 - 4x^2 - 10x - 36x^2 + 6x + 15$$

$$= 24x^3 - 40x^2 - 4x + 15$$

Question 4. *(4 marks)* Expand and simplify:

$$4x(3x-2)-2(x+5)(x-1)$$

$$= 12x^2 - 8x - 2(x^2 + 5x - x - 5)$$

$$= 12x^2 - 8x - 2(x^2 + 4x - 5)$$

$$= 12x^2 - 8x - 2x^2 - 8x + 10$$

$$= 10x^2 - 16x + 10$$

Question 5. *(4 marks)* Divide using long division and write what the following equals:

$$\frac{6x^2-13x+3}{3x-2}$$

$$
\begin{array}{r}
2x - 3 \\
3x-2\overline{\smash{)}\,6x^2-13x+3}
\end{array}
$$

$2x(3x-2) \rightarrow \quad -(6x^2 - 4x)$

$\qquad\qquad\qquad -9x + 3$

$-3(3x-2) \rightarrow \quad -(-9x+6)$

$\qquad\qquad\qquad\qquad -3$

$\therefore \quad \dfrac{6x^2-13x+3}{3x-2} = 2x-3 - \dfrac{3}{3x-2}$

Question 6. *(6 marks)* Divide using long division and state the quotient and remainder (indicate which is which):

$$\frac{4x^3 + x^4 - 3x + 5}{x^2 - 3x + 2}$$

$$
\begin{array}{r}
x^2 + 7x + 19 \\
x^2 - 3x + 2 \enclose{longdiv}{x^4 + 4x^3 + 0x^2 - 3x + 5}
\end{array}
$$

$x^2(x^2 - 3x + 2) \rightarrow \quad -(x^4 - 3x^3 + 2x^2) \downarrow$

$$7x^3 - 2x^2 - 3x$$

$7x(x^2 - 3x + 2) \longrightarrow -(7x^3 - 21x^2 + 14x) \downarrow$

$$19x^2 - 17x + 5$$

$19(x^2 - 3x + 2) \longrightarrow -(19x^2 - 57x + 38)$

$$40x - 33$$

QUOTIENT: $x^2 + 7x + 19$

REMAINDER: $40x - 33$

Question 7 Factor completely using the appropriate method. Check your answer when indicated.

(a) *(2 marks)*

$$x^4y^6 - x^3y^7 + x^5y^2 = x^3y^2(xy^4 - y^5 + x^2)$$

(b) *(3 marks)* (check your answer)

$$x^2 + 11x - 26 = (x + 13)(x - 2)$$

CHECK: $(x + 13)(x - 2)$

$= x^2 + 13x - 2x - 26$

$= x^2 + 11x - 26$

CASE 1

$A \cdot B = -26$ ⟹ $A = 13$

$A + B = 11$ $B = -2$

(c) *(3 marks)*

$$x^4 - 81 = (x^2)^2 - (9)^2 = (x^2 + 9)(x^2 - 9)$$

$$= (x^2 + 9)(x + 3)(x - 3)$$

(d) *(3 marks)* (check your answer)

$$27x^3 + 8 = (3x)^3 + 2^3$$

$$= (3x + 2)((3x)^2 - (3x)(2) + (2)^2)$$

$$= \boxed{(3x + 2)(9x^2 - 6x + 4)}$$

CHECK: $(3x + 2)(9x^2 - 6x + 4)$

$= 3x(9x^2 - 6x + 4) + 2(9x^2 - 6x + 4)$

$= 27x^3 - 18x^2 + 12x + 18x^2 - 12x + 8$

$= 27x^3 + 8$

NOTE $\sqrt{b^2 - 4ac}$

$= \sqrt{(-6)^2 - 4(4)(4)}$

$= \sqrt{-108}$

∴ CANNOT BE FACTORED ANY FURTHER

(e) *(3 marks)*

$$2x^2 - 15x + 18$$

$$= 2x^2 - 12x - 3x + 18$$

$$= 2x(x-6) - 3(x-6)$$

$$= (2x-3)(x-6)$$

CASE 2

$A \cdot B = 2(18) = 36$

$A + B = -15$

$A = -12 \quad B = -3$

(f) (3 marks)

$$x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3)$$

$$= (x-3)(x^2-4)$$

$$= (x-3)(x-2)(x+2)$$

(g) (3 marks)

$$7x^4 + 7x^3 - 140x^2 = 7x^2(x^2 + x - 20)$$

$$= 7x^2(x-4)(x+5)$$

CASE 1

$A \cdot B = -20$

$A + B = 1$

$A = -4, B = 5$

Question 8 Solve for x

BONUS

$9x^2 - 27 = 0$

$9(x^2 - 3) = 0$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

(b) *(4 marks)*

$7x^2 - 27x = 4$

$7x^2 - 27x - 4 = 0$

$7x^2 - 28x + x - 4 = 0$

$7x(x - 4) + (x - 4) = 0$

$(7x + 1)(x - 4) = 0$

$7x + 1 = 0 \qquad x - 4 = 0$

$7x = -1 \qquad\qquad x = 4$

$x = -1/7$

$\therefore x = \dfrac{-1}{7}, 4$

CASE 2

$A \cdot B = -4 \cdot 7 = 28$

$A + B = -27$

$A = -28 \quad B = 1$