

Last Name: SOLUTIONS
 First Name: _____
 Student ID: _____

Test 3

Question 1. Solve for x.

(a) (4 marks)

$$7x^3 + 47x^2 = 14x$$

$$7x^3 - 14x + 47x^2 = 0$$

$$7x^3 + 47x^2 - 14x = 0$$

$$x(7x^2 + 47x - 14) = 0$$

$$x(7x^2 + 49x - 2x - 14) = 0$$

$$x[7x(x+7) - 2(x+7)] = 0$$

$$x(7x-2)(x+7) = 0$$

$$\leftarrow x=0$$

$$\downarrow 7x-2=0$$

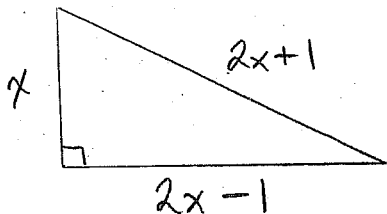
$$\rightarrow x+7=0$$

$$x = -7$$

$$7x = 2 \Rightarrow x = 2/7$$

$$\therefore x = -7, \frac{2}{7}, 0$$

(b) (4 marks)



$$x^2 + (2x-1)^2 = (2x+1)^2$$

$$x^2 + (2x-1)(2x-1) = (2x+1)(2x+1)$$

$$x^2 + 4x^2 - 2x - 2x + 1 = 4x^2 + 2x + 2x + 1$$

$$5x^2 - 4x + 1 = 4x^2 + 4x + 1$$

$$x^2 - 8x = 0$$

$$x(x-8) = 0$$

$$\leftarrow x=0$$

$$\downarrow x-8=0$$

$$x=8$$

LENGTH OF
 SIDES CAN'T BE
 NEGATIVE

$$\therefore \boxed{x=8}$$

Question 2. (4 marks) A girl is 5 years younger than her ^{SISTER} and the product of their ages is 204. Find their ages using a quadratic equation.

LET x BE SISTER'S AGE
 $x-5$ GIRL'S AGE

$$\therefore x(x-5) = 204$$

$$(x^2 - 5x = 204)$$

$$x^2 - 5x - 204 = 0$$

$$(x - 17)(x + 12) = 0$$

$$x - 17 = 0$$

$$x = 17$$

$$\vee x + 12 = 0$$

$$x = -12$$

CAN'T HAVE
 NEGATIVE AGE

$$\therefore x = 17$$

\(\therefore\) THE GIRL'S AGE IS 12
 AND HER SISTER IS 17

Question 3. (4 marks) Multiply and simplify:

$$\frac{x^2 + 7x - 30}{5x^2 - 3x} \cdot \frac{5x^2 + 2x - 3}{x^2 + 9x - 10} =$$

$$= \frac{(x+10)(x-3)}{x(5x-3)} \cdot \frac{(x+1)(5x-3)}{(x+10)(x-1)}$$

$$= \frac{(x-3)(x+1)}{x(x-1)}$$

$$\begin{aligned} & 5x^2 + 2x - 3 \\ &= 5x^2 + 5x - 3x - 3 \\ &= 5x(x+1) - 3(x+1) \\ &= (x+1)(5x-3) \end{aligned}$$

Question 4. (6 marks) Multiply, divide and simplify:

$$\frac{x^2-16}{x^2-x-42} \cdot \frac{x^2+5x+6}{x^2+8x+16} \div \frac{3x^2+8x+4}{3x^2-16x-12} = \frac{x^2-16}{x^2-x-42} \cdot \frac{x^2+5x+6}{x^2+8x+16} \cdot \frac{3x^2-16x-12}{3x^2+8x+4}$$

$$\frac{\cancel{(x+4)}(x-4)}{(x-7)(x+6)} \cdot \frac{(x+3)\cancel{(x+2)}}{\cancel{(x+4)}(x+4)} \cdot \frac{(3x+2)(x-6)}{\cancel{(3x+2)}(x+2)}$$

$$= \frac{(x-4)(x+3)(x-6)}{(x-7)(x+6)(x+4)}$$

Question 5. (5 marks) Subtract and simplify:

$$\frac{4}{x^2-6x-27} - \frac{2}{x^2-12x+27} = \frac{4}{(x+3)(x-9)} - \frac{2}{(x-3)(x-9)}$$

$$= \frac{4(x-3) - 2(x+3)}{(x+3)(x-9)(x-3)} = \frac{4x-12-2x-6}{(x+3)(x-9)(x-3)}$$

$$= \frac{2x-18}{(x+3)(x-9)(x-3)} = \frac{2(x-9)}{(x+3)\cancel{(x-9)}(x-3)}$$

$$= \frac{2}{(x+3)(x-3)}$$

Question 6. (5 marks) Simplify the complex fraction.

$$\frac{2 + \frac{10}{x}}{1 - \frac{25}{x^2}} = \frac{\frac{2x + 10}{x}}{\frac{x^2 - 25}{x^2}} = \frac{2x + 10}{x} \div \frac{x^2 - 25}{x^2}$$

$$= \frac{2x + 10}{x} \cdot \frac{x^2}{x^2 - 25} = \frac{2(x + 5)}{x} \cdot \frac{x^2}{(x + 5)(x - 5)} = \frac{2x}{x - 5}$$

Question 7. (5 marks) Last season a hockey player scored $\frac{1}{2}$ of his goals in the first period and $\frac{1}{3}$ in the second period. How many goals did he score in the last season in total if he scored 9 goals in the third period?

LET x BE THE NUMBER OF GOALS HE SCORED LAST SEASON

$$\begin{array}{c} \frac{1}{2}x + \frac{1}{3}x + 9 = x \leftarrow \text{TOTAL GOALS} \quad \text{LCD} = 6 \\ \begin{array}{ccc} \nearrow & \uparrow & \nwarrow \\ \text{FIRST PERIOD} & \text{SECOND} & \text{THIRD} \\ \text{GOALS} & \text{PERIOD GOALS} & \text{PERIOD GOALS} \end{array} \end{array}$$

$$6 \cdot \frac{1}{2}x + 6 \cdot \frac{1}{3}x + 6 \cdot 9 = 6x$$

$$3x + 2x + 54 = 6x$$

$$54 = x$$

\(\therefore\) HE SCORED 54 GOALS

Questions 8. Use the LCD to solve for x:
(a) (5 marks)

$$1 + \frac{5}{x-4} = \frac{8x+13}{x^2+x-20}$$

$$1 + \frac{5}{x-4} = \frac{8x+13}{(x+5)(x-4)}$$

$$\text{LCD} = (x+5)(x-4)$$

$$(x+5)(x-4) + 5(x+5) = 8x+13$$

$$x^2 + x - 20 + 5x + 25 = 8x + 13$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

↓

$$x = 4$$

↓

$$x = -2 \text{ VALID}$$

EXTRANEJOS

$$\therefore \boxed{x = -2}$$

(b) (5 marks)

$$\frac{2}{2+x} + \frac{1}{2-x} = \frac{10}{x^2-4}$$

$$\frac{2}{x+2} - \frac{1}{x-2} = \frac{10}{(x+2)(x-2)}$$

$$\text{LCD} = (x+2)(x-2)$$

$$2(x-2) - (x+2) = 10$$

$$2x - 4 - x - 2 = 10$$

$$x - 6 = 10$$

$$x = 16 \text{ VALID}$$

$$\therefore \boxed{x = 16}$$

Question 9. (3 marks) Solve for x in the proportion:

$$\frac{7}{3x+1} = \frac{4}{x-2}$$

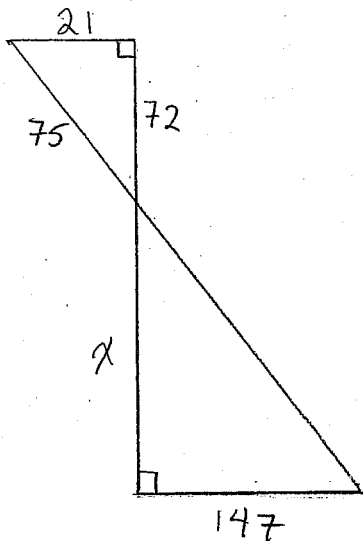
$$7(x-2) = 4(3x+1)$$

$$7x - 14 = 12x + 4$$

$$-18 = 5x$$

$$\therefore x = \frac{-18}{5}$$

Question 10. (3 marks) The following are similar triangles. Solve for x .



$$\frac{x}{147} = \frac{72}{21}$$

$$21x = 147(72)$$

$$x = \frac{147(72)}{21} = 504$$

Question 11. (4 marks) Two numbers are in the ratio of 5:8. If 9 is added to each number the resulting ratio is 8:11. Find the numbers.

LET THE NUMBERS BE $5x$ AND $8x$

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$\Rightarrow 11(5x+9) = 8(8x+9)$$

$$55x+99 = 64x+72$$

$$27 = 9x$$

$$3 = x$$

\therefore THE NUMBERS ARE 15 AND 24

Question 12 Simplify the following:

(a) (2 marks)

$$5\sqrt{6} \cdot 2\sqrt{24} = 10\sqrt{144} = 10 \cdot 12 = 120$$

(b) (2 marks)

$$\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$$

Question 13 Multiply and simplify:

(a) (4 marks)

$$\begin{aligned}(3\sqrt{2}-4\sqrt{3})^2 &= (3\sqrt{2}-4\sqrt{3})(3\sqrt{2}-4\sqrt{3}) \\ &= 9 \cdot 2 - 12\sqrt{6} - 12\sqrt{6} + 16 \cdot 3 \\ &= 18 - 24\sqrt{6} + 48 \\ &= 66 - 24\sqrt{6}\end{aligned}$$

(b) (4 marks)

$$\begin{aligned}(3\sqrt{5}-4\sqrt{2})(2\sqrt{5}+3\sqrt{2}) \\ &= 6 \cdot 5 + 9\sqrt{10} - 8\sqrt{10} - 12 \cdot 2 \\ &= 30 + \sqrt{10} - 24 \\ &= 6 + \sqrt{10}\end{aligned}$$