

Quiz 6

Question 1. (10 marks) Solve the following system using the inverse of the coefficient matrix.

$$\begin{array}{r}
 x + 2y + 2z = -4 \\
 4x + 9y + 10z = -18 \\
 -x + 3y + 7z = -7
 \end{array}
 \Leftrightarrow
 \begin{array}{c}
 A \cdot X = C \\
 \begin{bmatrix} 1 & 2 & 2 \\ 4 & 9 & 10 \\ -1 & 3 & 7 \end{bmatrix}
 \begin{bmatrix} x \\ y \\ z \end{bmatrix}
 =
 \begin{bmatrix} -4 \\ -18 \\ -7 \end{bmatrix}
 \end{array}$$

$$\begin{array}{c}
 \begin{bmatrix} 1 & 2 & 2 & | & 1 & 0 & 0 \\ 4 & 9 & 10 & | & 0 & 1 & 0 \\ -1 & 3 & 7 & | & 0 & 0 & 1 \end{bmatrix}
 \xrightarrow[\begin{array}{l} R_2 - 4R_1 \\ R_3 + R_1 \end{array}]{A}
 \begin{bmatrix} 1 & 2 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -4 & 1 & 0 \\ 0 & 5 & 9 & | & 1 & 0 & 1 \end{bmatrix}
 \xrightarrow[\begin{array}{l} R_1 - 2R_2 \\ R_3 - 5R_2 \end{array}]{I}
 \begin{bmatrix} 1 & 0 & -2 & | & 9 & -2 & 0 \\ 0 & 1 & 2 & | & -4 & 1 & 0 \\ 0 & 0 & -1 & | & 21 & -5 & 1 \end{bmatrix}
 \end{array}$$

$$\begin{array}{c}
 \begin{bmatrix} 1 & 0 & -2 & | & 9 & -2 & 0 \\ 0 & 1 & 2 & | & -4 & 1 & 0 \\ 0 & 0 & 1 & | & -21 & 5 & -1 \end{bmatrix}
 \xrightarrow[\begin{array}{l} R_1 + 2R_3 \\ R_2 - 2R_3 \end{array}]{I}
 \begin{bmatrix} 1 & 0 & 0 & | & -33 & 8 & -2 \\ 0 & 1 & 0 & | & 38 & -9 & 2 \\ 0 & 0 & 1 & | & -21 & 5 & -1 \end{bmatrix}
 \xrightarrow{R_3 - (-1)}{A^{-1}}
 \end{array}$$

$$X = A^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}
 =
 \begin{bmatrix} -33 & 8 & -2 \\ 38 & -9 & 2 \\ -21 & 5 & -1 \end{bmatrix}
 \begin{bmatrix} -4 \\ -18 \\ -7 \end{bmatrix}
 =
 \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$$

$$\therefore x=2, y=-4, z=1$$