

SOLUTIONS TO CIRCLE LINE PROBLEMS

$$1) \quad y = mx + b$$

$$-\frac{625}{24} = \frac{7}{24}(0) + b$$

$$-\frac{625}{24} = b$$

$$y = \frac{7}{24}x - \frac{625}{24}$$

$$x^2 + y^2 = 625$$

$$x^2 + \left(\frac{7x - 625}{24}\right)^2 = 625$$

$$x^2 + \frac{49x^2}{576} - \frac{4375x}{576} - \frac{4375x}{576} + \frac{390625}{576} = 625$$

$$576x^2 + 49x^2 - 4375x - 4375x + 390625 = 360000$$

$$625x^2 - 8750x + 30625 = 0$$

$$625(x^2 - 14x + 49) = 0$$

$$x^2 - 14x + 49 = 0$$

$$(x - 7)(x - 7) = 0$$

$$\therefore x = 7$$

FIND y , PLUG INTO $y = \frac{7}{24} \cdot 7 - \frac{625}{24}$

$$= \frac{49}{24} - \frac{625}{24} = \frac{-576}{24}$$

$$= -24$$

\therefore THE POINT OF INTERSECTION IS $(7, -24)$

\therefore IT IS A TANGENT LINE

$$2) m = \frac{3}{4}$$

$$y = mx + b$$

$$\frac{9}{4} = \frac{3}{4}(0) + b$$

$$b = \frac{9}{4} \Rightarrow y = \frac{3}{4}x + \frac{9}{4}$$

INTERSECTION OF LINES AND CIRCLES:

$$x^2 + (y - 4)^2 = 25$$

$$x^2 + \left(\frac{3}{4}x + \frac{9}{4} - 4\right)^2 = 25$$

$$x^2 + \left(\frac{3}{4}x - \frac{7}{4}\right)^2 = 25$$

$$x^2 + \left(\frac{3}{4}x - \frac{7}{4}\right)\left(\frac{3}{4}x - \frac{7}{4}\right) = 25$$

$$x^2 + \frac{9}{16}x^2 - \frac{21}{16}x - \frac{21}{16}x + \frac{49}{16} = 25$$

$$16x^2 + 9x^2 - 21x - 21x + 49 = 400$$

$$25x^2 - 42x - 351 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-42) \pm \sqrt{(-42)^2 - 4(25)(-351)}}{2(25)}$$

$$= \frac{42 \pm \sqrt{36864}}{50} = \frac{42 \pm 192}{50}$$

$$\therefore x = \frac{42 + 192}{50} = \frac{234}{50} = \frac{117}{25} \quad x = \frac{42 - 192}{50} = -3$$

FIND y , PLUG INTO $y = \frac{3}{4}x + \frac{9}{4}$

$$y = \frac{3}{4}(-3) + \frac{9}{4} = 0$$

$$\therefore (-3, 0)$$

$$y = \frac{3}{4} \left(\frac{117}{25} \right) + \frac{9}{4} = \frac{351}{100} + \frac{255}{100} = \frac{576}{100} = \frac{144}{25}$$

$$\therefore \left(\frac{117}{25}, \frac{144}{25} \right)$$

\(\therefore\) THE POINTS OF INTERSECTION ARE

$$(-3, 0) \text{ AND } \left(\frac{117}{25}, \frac{144}{25} \right)$$

\(\therefore\) IT IS A SLOANT LINE

$$3) \quad 13y - 2x = 27$$

$$13y = 2x + 27$$

$$y = \frac{2}{13}x + \frac{27}{13} \Rightarrow m = 2/13$$

$$y = mx + b$$

$$16 = \frac{2}{13}(14) + b$$

$$16 = \frac{28}{13} + b$$

$$\frac{180}{13} = b \Rightarrow y = \frac{2}{13}x + \frac{180}{13}$$

$$\text{INTERSECTIONS: } (x+3)^2 + (y-1)^2 = 169$$

$$(x+3)^2 + \left(\frac{2}{13}x + \frac{180}{13} - 1 \right)^2 = 169$$

$$x^2 + 6x + 9 + \left(\frac{2}{13}x + \frac{167}{13} \right)^2 = 169$$

$$x^2 + 6x + 9 + \frac{4}{169}x^2 + \frac{334}{169}x + \frac{334}{169}x + \frac{27889}{169} = 169$$

$$169x^2 + 1014x + 1521 + 4x^2 + 334x + 334x + 27889 = 28561$$
$$173x^2 + 1682x + 849 = 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1682 \pm \sqrt{(1682)^2 - 4(173)(849)}}{2(173)}$$
$$= \frac{-1682 \pm \sqrt{2241616}}{346}$$

$$\therefore y = \frac{2}{13} \left(\frac{-1682 + \sqrt{2241616}}{346} \right) + \frac{27}{13}$$

AND

$$y = \frac{2}{13} \left(\frac{-1682 - \sqrt{2241616}}{346} \right) + \frac{27}{13}$$

\(\therefore\) THE INTERSECTIONS ARE

$$\left(\frac{-1682 + \sqrt{2241616}}{346}, \frac{2}{13} \left(\frac{-1682 + \sqrt{2241616}}{346} \right) + \frac{27}{13} \right)$$

AND

$$\left(\frac{-1682 - \sqrt{2241616}}{346}, \frac{2}{13} \left(\frac{-1682 - \sqrt{2241616}}{346} \right) + \frac{27}{13} \right)$$

\(\therefore\) SLOANT LINE

$$4) 5y = -4x - 15 \Rightarrow y = \frac{-4x - 15}{5}$$

$$\therefore \text{OUR SLOPE } m = \frac{5}{4}$$

$$y = mx + b$$

$$3 = \frac{5}{4}(4) + b$$

$$-2 = b$$

$$\therefore y = \frac{5x}{4} - 2$$

INTERSECTION:

$$(x-1)^2 + (y+1)^2 = 25$$

$$(x-1)^2 + \left(\frac{5x-2}{4} + 1\right)^2 = 25$$

$$x^2 - 2x + 1 + \left(\frac{5x-1}{4}\right)^2 = 25$$

$$x^2 - 2x + 1 + \frac{25x^2}{16} - \frac{5x}{4} - \frac{5x}{4} + 1 = 25$$

$$x^2 - 2x + 1 + \frac{25x^2}{16} - \frac{10x}{4} + 1 = 25$$

$$16x^2 - 32x + 32 + 25x^2 - 40x + 16 = 400$$

$$41x^2 - 72x - 352 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-72) \pm \sqrt{(-72)^2 - 4(41)(-352)}}{2(41)}$$

$$= \frac{72 \pm \sqrt{62912}}{82} = \frac{72 \pm 8\sqrt{983}}{82} = \frac{36 \pm 4\sqrt{983}}{41}$$

$$y = \frac{5}{4} \left(\frac{36 \pm 4\sqrt{983}}{41} \right) - 2$$

∴ INTERCEPTS:

$$\left(\frac{36 + 4\sqrt{983}}{41}, \frac{5}{4} \left(\frac{36 + 4\sqrt{983}}{41} \right) - 2 \right)$$

AND

$$\left(\frac{36 - 4\sqrt{983}}{41}, \frac{5}{4} \left(\frac{36 - 4\sqrt{983}}{41} \right) - 2 \right)$$

∴ SECANT LINE