# Test 2

**Question 1.** *(10 marks)*
Given that $A$ is a $3 \times 3$ matrix and

$$B=\begin{bmatrix} 1 & -4 & 0 \\ 2 & 1 & 4 \end{bmatrix} \quad C=\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 5 & -2 & -1 \\ 0 & 1 & 3 \end{bmatrix} \quad E=\begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 7 & -4 \end{bmatrix}$$

find the following if possible. If not possible explain why.
**(a)** $CD$

$$CD=\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 5 & -2 & -1 \\ 0 & 1 & 3 \end{bmatrix}=\begin{bmatrix} (-1)(5)+(2)(0) & (-1)(-2)+(2)(1) & (-1)(-1)+(2)(3) \\ (-3)(5)+(1)(0) & (-3)(-2)+(1)(1) & (-3)(-1)+(1)(3) \end{bmatrix}$$

$$=\begin{bmatrix} -5 & 4 & 7 \\ -15 & 7 & 6 \end{bmatrix}$$

**(b)** $\frac{1}{2}E(C-D)$ IMPOSSIBLE
DIFFERENT SIZES

(c) $DE - \frac{1}{5}C$ $= \begin{bmatrix} 5 & -2 & -1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 7 & -4 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3 & 7 \\ 24 & -11 \end{bmatrix} - \begin{bmatrix} -1/5 & 2/5 \\ -3/5 & 1/5 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} & \frac{33}{5} \\ \frac{123}{5} & \frac{-56}{5} \end{bmatrix}$

(d) $\underbrace{A^{-1}A}_{I}EB$ $= IEB = EB$

$= \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 7 & -4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -7 & 4 \\ 5 & -11 & 4 \\ -1 & -32 & -16 \end{bmatrix}$

(d) $C^{-1}B$ $C^{-1} = \frac{1}{(-1)(1)-(-3)(2)} \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/5 & \frac{-2}{5} \\ 3/5 & -1/5 \end{bmatrix}$

$C^{-1}B = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} \\ \frac{3}{5} & \frac{-1}{5} \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{-3}{5} & \frac{-6}{5} & \frac{-8}{5} \\ \frac{1}{5} & \frac{-13}{5} & \frac{-4}{5} \end{bmatrix}$

**Question 2.** *(5 marks)* An alloy used in electrical transformers contains nickel (Ni), Iron (Fe) end Molybdenum (Mo). The percent of Ni is 1% less than five times the percent of Fe. The percent of Fe is 1% more than three times the percent of Mo. Find the percent of each in the alloy.

Fe, Mo, Ni - PERCENTAGE OF EACH ELEMENT

$\therefore$ $Mo + Fe + Ni = 100$ $\quad\Rightarrow\quad$ $Mo + Fe + Ni = 100$ ①

$Ni = 5Fe - 1$ $\quad\Rightarrow\quad$ $-5Fe + Ni = -1$ ②

$Fe = 3Mo + 1$ $\quad\Rightarrow\quad$ $-3Mo + Fe = 1$ ③

① $Mo + Fe + Ni = 100$

② $-(-5e + Ni = -1)$

① $Mo + 6Fe = 101$

③ $-3Mo + Fe = 1$

$3 \times$ ① $+ (3Mo + 18Fe = 303)$

$19Fe = 304$

$Fe = 16$

$-3Mo + 16 = 1$

$-3Mo = -15$

$Mo = 5$

$\Rightarrow$ $5 + 16 + Ni = 100$

$\therefore Ni = 79$

$\therefore$ THE ALLOY HAS 16% Fe, 5% Mo AND 79% Ni

**Question 3.** *(6 marks)* Solve for $x$ and $y$:

$$\begin{bmatrix} x+2y \\ 3x+5y \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 24 \end{bmatrix}$$

$\Rightarrow$ $x + 2y = 14$

$3x + 5y = 24$

$\Rightarrow$ $3x + 6y = 42$

$-(3x + 5y = 24)$

$y = 18$

$3x + 6(18) = 42$

$3x + 108 = 42$

$3x = -66$

$x = -22$

$\therefore x = -22, \quad y = 18$

**Question 4.** *(8 marks)*

**(a)** Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -5 \\ -1 & -2 & 0 \end{bmatrix},$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -5 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+R_1]{R_2-3R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -3 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 7 & -2 & 0 \\ 0 & 1 & -2 & -3 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \cdot (-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 7 & -2 & 0 \\ 0 & 1 & -2 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right] \xrightarrow[R_2+2R_3]{R_1-3R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -2 & 3 \\ 0 & 1 & 0 & -5 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right] \quad \therefore A^{-1} = \begin{bmatrix} 10 & -2 & 3 \\ -5 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

**(b)** Write the following system as a matrix equation and use part **(a)** to solve the system:

$$\begin{array}{rcrcrcl} x & + & 2y & - & z & = & 1 \\ 3x & + & 7y & - & 5z & = & -3 \\ -x & - & 2y & & & = & 2 \end{array} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -5 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$$

$$A \cdot X = B \Rightarrow X = A^{-1}B.$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 & -2 & 3 \\ -5 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 22 \\ -12 \\ -3 \end{bmatrix}$$

$$\therefore x = 22, \quad y = -12, \quad z = -3$$

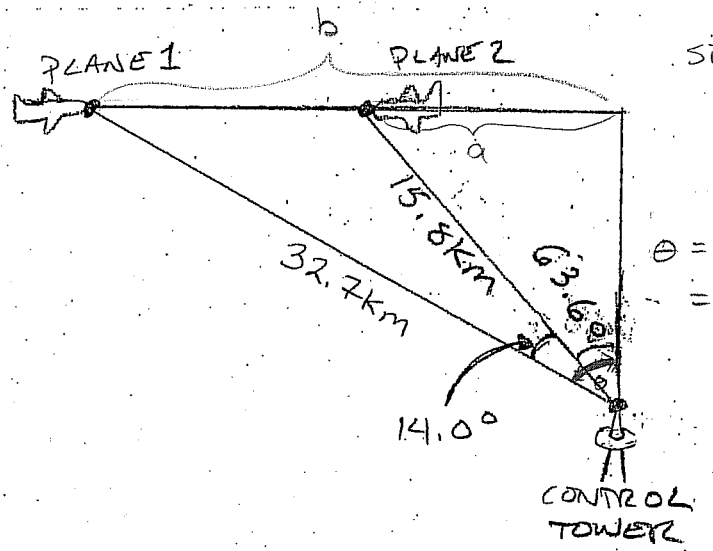
**Question 5.** *(4 marks)* Determine whether of not $A = B^{-1}$ (justify your answer)

$$A = \begin{bmatrix} 1/2 & 3/4 \\ 3/2 & -1/2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1/2 & 3/4 \\ 3/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 1/2 + \frac{9}{4} & -1 - \frac{18}{4} \\ 3/2 - 3/2 & -\frac{6}{2} + \frac{6}{2} \end{bmatrix} = \begin{bmatrix} \frac{11}{4} & -\frac{11}{2} \\ 0 & 0 \end{bmatrix} \neq I$$

$\therefore A \neq B^{-1}$

**Question 6.** *(6 marks)* An air traffic controller sights two planes that are due east from the control tower and headed toward each other. One is 15.8km from the tower and the other is 32.7km from the tower as in the diagram below. How far apart are the planes?




$\sin 63.6° = \frac{a}{15.8} \Rightarrow a = 15.8 \sin 63.6°$
$= 14.2$

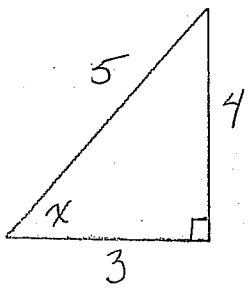
$\theta = 63.6 + 14.0$
$= 77.6°$

$\sin 77.6° = \frac{b}{32.7} \Rightarrow b = 32.7 \sin 77.6°$
$= 31.9$

$\therefore$ DISTANCE BETWEEN PLANES: $31.9 - 14.2$
$= 17.7\text{km}$

**Question 7.** *(7 marks)*

(a) Given the following find $\sec x$, $\csc x$ and $\cot x$:

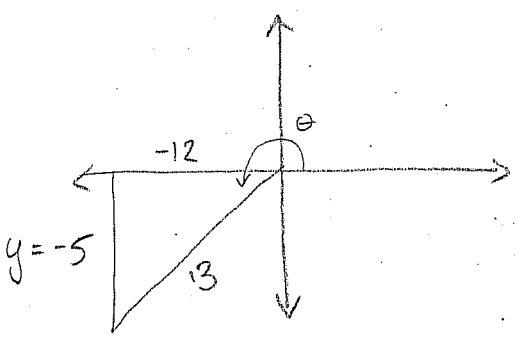



$\sec x = \frac{5}{3}$

$\csc x = \frac{5}{4}$

$\cot x = \frac{3}{4}$

**(b)** Given $\cos\theta = -12/13$ and $\theta$ is in the third quadrant use the appropriate right triangle to find $\sin\theta$, $\tan\theta$ and $\sec\theta$.




$(-12)^2 + y^2 = 13^2$

$y^2 = 13^2 - 12^2 = 25$

$y = -\sqrt{25} = -5$

↑ THIRD QUADRANT

$\therefore \sin\theta = \frac{-5}{13}$, $\tan\theta = \frac{-5}{-12} = \frac{5}{12}$

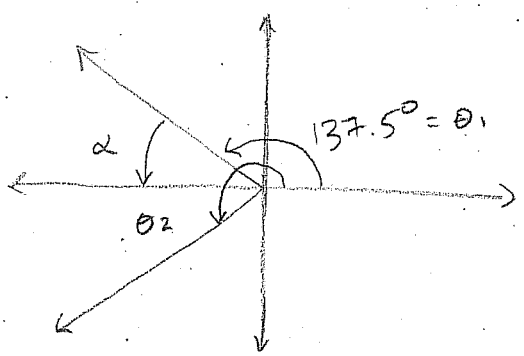
$\sec\theta = -\frac{13}{12}$

**Question 8.** *(6 marks)* Find $\theta$ for $0° \le \theta \le 360°$ (Round correctly):

**(a)** $\cos\theta = -0.737$

$\cos^{-1}(-0.737) = 137.5°$

$\theta$ IS IN 2nd OR 3rd QUADRANT




$\alpha = 180° - 137.5° = 42.5°$

$\theta_2 = 180° + 42.5° = 222.5°$

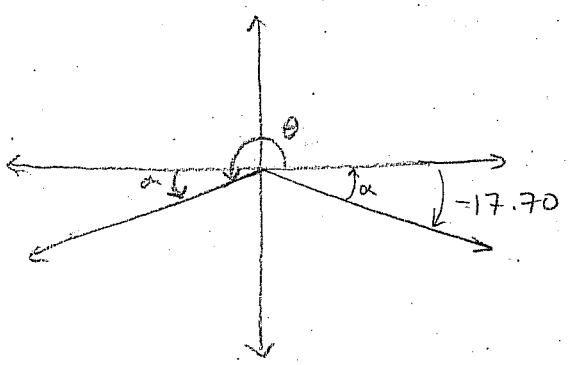
$\therefore \quad \theta = 137.5°$ OR $222.5°$

**(b)** $\sin\theta = -0.3040$ and $\tan\theta > 0$

$\sin^{-1}(-0.3040) = -17.70°$

$\theta$ IS IN 3rd QUADRANT ($\sin\theta < 0$, $\tan\theta > 0$)

$\alpha = 17.70°$




$\theta = 180° + 17.70$

$= 197.70°$

**Bonus.** *(2 marks)* Solve for $x$:

$$\begin{vmatrix} 3 & 0 & -1 \\ 1 & x & 2 \\ 1 & 2 & x+1 \end{vmatrix} = \begin{vmatrix} 2 & x \\ -1 & x+2 \end{vmatrix}$$

$\begin{vmatrix} 2 & x \\ -1 & x+2 \end{vmatrix} = 2x+4+x = 3x+4$

$\begin{vmatrix} 3 & 0 & -1 \\ 1 & x & 2 \\ 1 & 2 & x+1 \end{vmatrix} \begin{matrix} 3 & 0 \\ 1 & x \\ 1 & 2 \end{matrix}$

$= 3x(x+1) + 0 - 2 + x - 12$

$= 3x^2 + 3x + x - 14$

$= 2x^2 + 4x - 14$

$\therefore \quad 3x^2 + 4x - 14 = 3x + 4 \Rightarrow 3x^2 + x - 18 = 0$

$\therefore \quad x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(3)(-18)}}{2(3)} = \dfrac{-1 \pm \sqrt{217}}{6}$