Dawson College: Chem Tech / Lab Tech: 201-921-DW 01

November 20, 2009

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 3

Question 1. *(3 marks)* find the following expressing your answers without decimals:

(a) $\sin \frac{\pi}{4}$

(b) $\cos 150°$

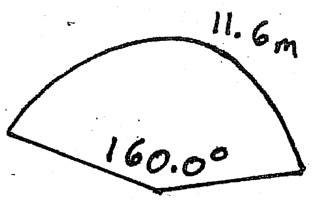
(c) $\tan 30°$

a) $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

b) $\cos 150° = -\cos 30° = -\sqrt{3}/2$

c) $\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{1/2}{\sqrt{3}/2} = 1/\sqrt{3}$

Question 2. *(6 marks)* A patio is in the shape of a circular sector with a central angle of 160.0°. It is enclosed by a railing of which the circular part is 11.6m long. What is the area of the patio?



$\theta = 160°\left(\frac{\pi}{180°}\right)$

$s = \theta r$

$11.6 = 160°\left(\frac{\pi}{180°}\right) r$

$\therefore r = 4.15\text{m}$

$A = \frac{1}{2}\theta r^2$

$= \frac{1}{2}\,160°\left(\frac{\pi}{180°}\right)(4.15)^2$

$= 24.0\text{m}^2$

Question 3. *(5 marks)* What is the diameter of a drill bit in mm that rotates at 1200r/min if a point on the circumference has linear velocity 35.9m/min?

$\omega = (1200r/min)(2\pi\ rad) = 2400\ rad/min$

$v = \omega r$

$35.9 = 2400\pi r$

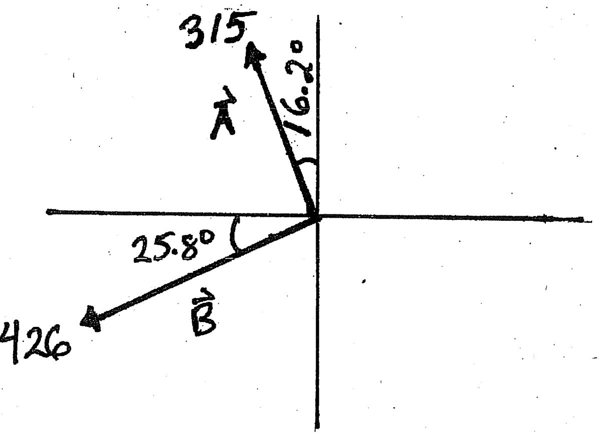
$r = 0.004761 39m$

$= 4.76139mm$

$\therefore d = 2r$

$= 9.5mm$

Question 4. *(6 marks)* Add the following vectors using their components to find the resulting vector.



$\theta_A = 90 + 16.2 = 106.2°$

$A_x = A\cos\theta_A = 315\cos 106.2°$

$= -87.9$

$A_y = A\sin\theta_A = 315\sin 106.2°$

$= 302$

$\theta_B = 180 + 25.8 = 205.8°$

$B_x = B\cos\theta_B = 426\cos 205.8°$

$= -384$

$B_y = B\sin\theta_B = 426\sin 205.8°$

$= -185$

LET $\vec{R} = \vec{A} + \vec{B}$

$R_x = A_x + B_x = -87.9 - 384 = -472$

$R_y = A_y + B_y = 302 - 185 = 117$

$\therefore R = \sqrt{R_x^2 + R_y^2}$

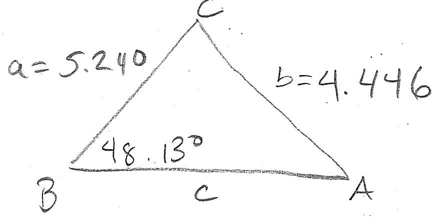
$= \sqrt{(-472)^2 + (117)^2}$

$= 486$

$\tan\theta_r = \frac{R_y}{R_x} = \frac{117}{-472} = -0.2478813559$

$\tan^{-1}\left(\frac{R_y}{R_x}\right) = -13.9°$ $\therefore \theta_R = 180 - 13.9 = 166.1°$

Question 5. *(6 marks)* Solve the triangle with sides $a = 5.240$, $b = 4.446$, and angle $B = 48.13°$.



$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin A = \frac{a \sin B}{b} = \frac{5.240 \sin 48.13°}{4.446}$$
$$= 0.8776482842$$
$$\sin^{-1}(0.8776482842)$$
$$= 61.36°$$

$\therefore$ TWO SOLUTIONS

$A = 61.36°$

$C = 180° - 61.36° - 48.13°$
$= 70.51°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c = \frac{b \sin C}{\sin B} = \frac{4.446 \sin 70.51°}{\sin 48.13°}$$

$= 5.628$

SOLUTION 1

$a = 5.240$

$b = 4.446$

$c = 5.628$

$A = 61.36°$

$B = 48.13°$

$C = 70.51°$

$A = 180 - 61.36 = 118.64°$

$C = 180° - 118.64° - 48.13°$
$= 13.23°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c = \frac{b \sin C}{\sin B} = \frac{4.446 \sin 13.23°}{\sin 48.13°}$$

$= 1.366$

SOLUTION 2

$a = 5.240$

$b = 4.446$

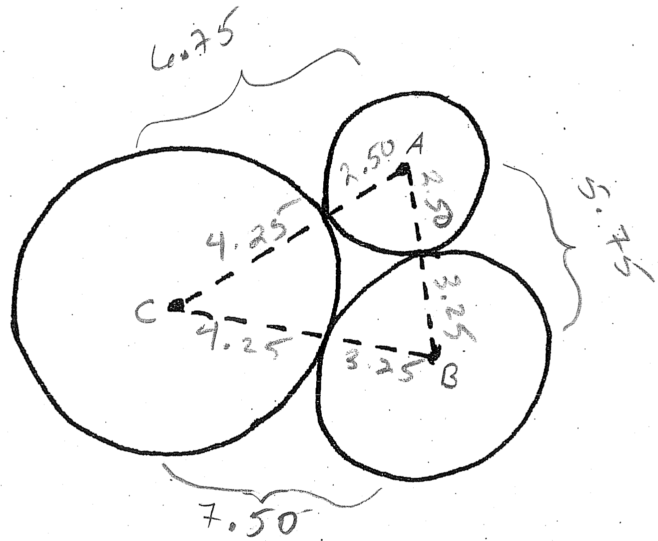
$c = 1.366$

$A = 118.64°$

$B = 48.13°$

$C = 13.23°$

Question 6. *(7 marks)* Three circular pipes with radii 2.50cm, 3.25cm and 4.25cm are welded together lengthwise (see the figure below). Find the angles between the center to center lines.



FIND LARGEST ANGLE

$a^2 = b^2 + c^2 + bc \cos A$

$7.50^2 = 6.75^2 + 5.75^2 - 2(6.75)(5.75)\cos A$

$\cos A = \dfrac{7.50^2 - 6.75^2 - 5.75^2}{-2(6.75)(5.75)} = 0.2882447665$

$A = 73.2°$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C} \Rightarrow \sin C = \dfrac{c \sin A}{a} = \dfrac{5.75 \sin 73.2°}{7.50} = 0.733944948$

$C = \sin^{-1}(0.7339449481) = 47.2°$

$C = 180° - 73.2° - 47.2° = 59.6°$

Question 7. *(6 marks)* Find the period, displacement and ~~phase shift~~ amplitude of

$$y = -6\sin\left(\frac{1}{4}x + \frac{\pi}{2}\right)$$

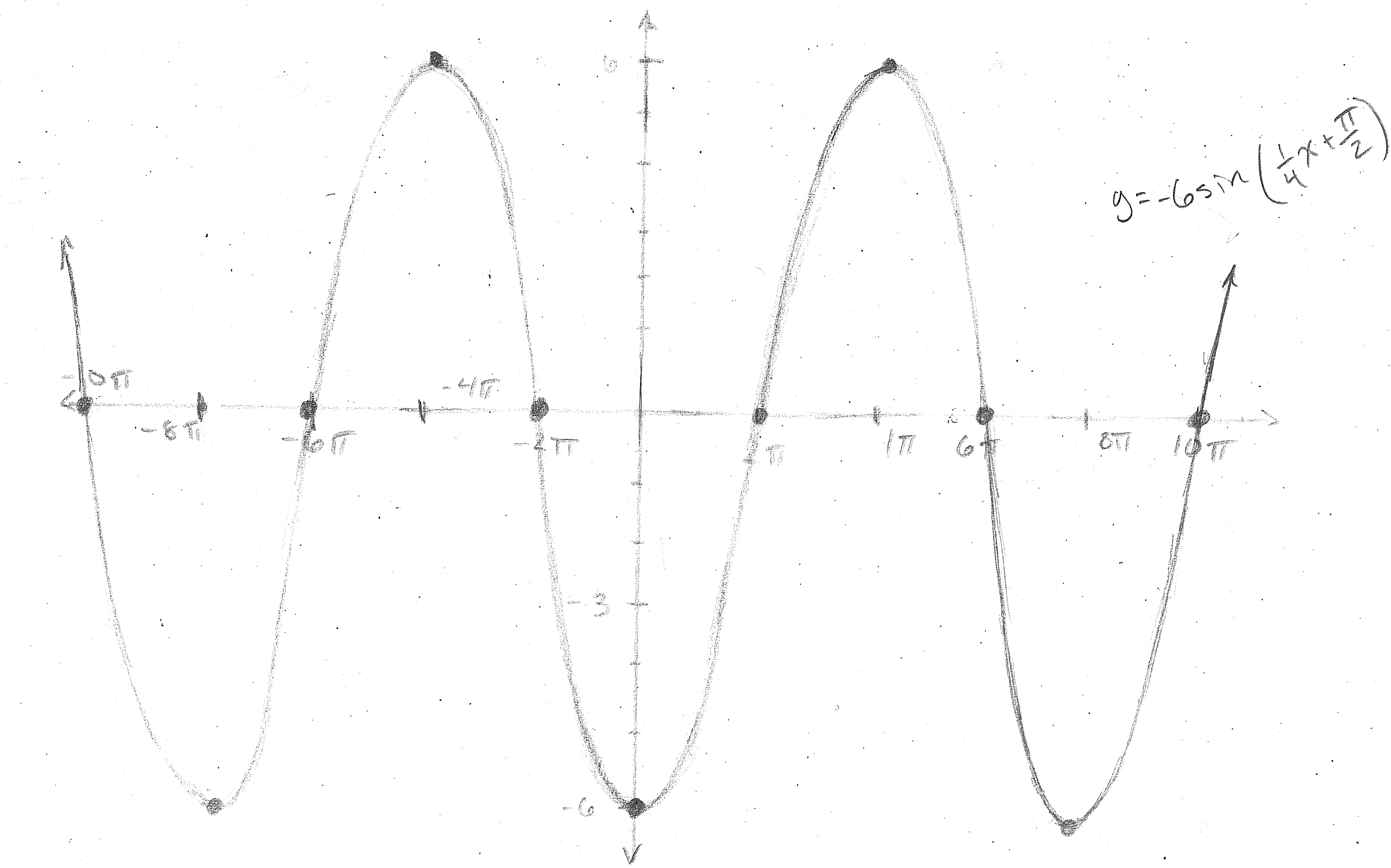
and clearly graph this function. Show at least one period to the right and to the left of the y-axis.

PERIOD: $\dfrac{2\pi}{1/4} = 8\pi$ $\longrightarrow$ INCREMENTS: $\dfrac{8\pi}{4} = 2\pi$

DISPLACEMENT: $\dfrac{-\pi/2}{1/4} = -2\pi$

AMPLITUDE: $|-6| = 6$

x	-2π	0	2π	4π	8π
y	0	-6	0	6	0



Question 8. *(7 marks)* Verify the following identities:

(a) $\tan x + \cot x = \tan x \csc^2 x$

$$LHS = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$$

$$RHS = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos x \sin x}$$

$\therefore LHS = RHS$

(b) $\dfrac{\csc\theta - 1}{\cot\theta} = \dfrac{\cot\theta}{\csc\theta + 1}$

$$LHS = \frac{\frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta}} = \frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} \cdot \frac{(1+\sin\theta)}{(1+\sin\theta)} = \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos\theta}{1+\sin\theta}$$

$$RHS = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} + 1} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1+\sin\theta}{\sin\theta}} = \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta}$$

$\therefore LHS = RHS$

Bonus. *(3 marks)* Given

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \quad \text{and} \quad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

show that

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

(hint: first find $\sin 2\alpha$ and $\cos 2\alpha$.)

$\sin 2\alpha = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$

$$\therefore \tan 2\alpha = \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha} \cdot \frac{\frac{1}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}$$

$$= \frac{\frac{2\sin\alpha\cancel{\cos}\alpha}{\cos^{\cancel{2}}\alpha}}{\frac{\cos^2\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha}}$$

$$= \frac{2\,\frac{\sin\alpha}{\cos\alpha}}{1 - \left(\frac{\sin\alpha}{\cos\alpha}\right)^2}$$

$$= \frac{2\tan\alpha}{1-\tan^2\alpha}$$