Test 1

Question 1. *(7 marks)*
(a) Simplify using positive exponents only:

$$\frac{(4ab)^{-2}}{2a^2b^{1/2}} \cdot \left(\frac{a^2}{b^{-1/2}}\right)^9$$

(b) Solve for p_2 in the following formula:

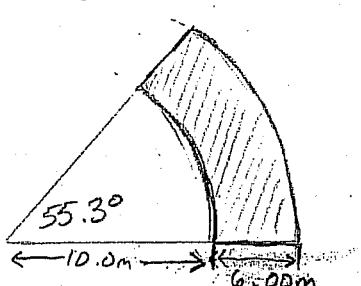
$$R = \frac{\pi r^4 (p_2 - p_1)}{8nL}$$

Question 2. *(8 marks)*
(a) Find the equation of the line passing through the point $(-4, 1)$ and perpendicular to the line $5x - 2y = 6$. Graph $5x - 2y = 6$.
(b) Find the radius and the centre of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$. Graph this circle.

Question 3. *(11 marks)* The weight of gold is 1206 pounds per cubic foot (1206 lb / ft^3).
(a) Convert this weight to kg per cubic cm.
(b) Convert this weight to pounds per cubic inch.
(c) Covert 100 km/ hour to feet / second.
(d) Convert 10 000 pounds per square inch (psi) to Mpa.

Question 4. *(7 marks)*
(a) Given $\csc\theta = 1.615$ find $\sin\theta$ and $\tan\theta$.
(b) Given $\cos\theta = -0.7963$ find θ for $0 \leq \theta \leq 360°$ and $\tan\theta > 0$.

Question 5. *(11 marks)*
(a) The following is a diagram of a banked turn on a road (the road is shaded). What is the area of the road? What is the length of the guard rail on the outer arc of the road?



(b) The paddles of a riverboat have a radius of 2.59m and revolve in a circle at 20.0 r/min (revolutions per minute). What is the speed of the tip of one of the paddles.

Question 6. *(6 marks)* Find $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ (magnitude and direction) given $A = 432$ $\theta_A = 105.1°$, and $B = 117$ $\theta_B = 148.2°$.

TEST 1 SOLUTIONS

a) $\frac{(4ab)^{-2}}{2a^2b^{\frac{1}{2}}} \cdot \left(\frac{a^2}{b^{-\frac{1}{2}}}\right)^9 = \frac{4^{-2}a^{-2}b^{-2}}{2a^2b^{\frac{1}{2}}} \cdot \frac{a^{18}}{b^{-\frac{9}{2}}} = \frac{a^{18}b^{\frac{9}{2}}}{2a^2b^{\frac{1}{2}} \cdot 4^2a^2b^2}$

$= \frac{a^{18}b^{\frac{9}{2}}}{32a^4b^{\frac{5}{2}}} = \frac{a^{14}b^{\frac{4}{2}}}{32} = \frac{a^{14}b^2}{32}$

b) $R = \frac{\pi r^4(p_2-p_1)}{8\eta L} \Rightarrow 8\eta LR = \pi r^4(p_2-p_1)$

$\Rightarrow \frac{8\eta LR}{\pi r^4} = p_2-p_1 \Rightarrow \frac{8\eta LR}{\pi r^4} + p_1 = p_2$

2) a) $5x-2y=6 \Rightarrow -2y = -5x+6 \Rightarrow y = \frac{5}{2}x - 3$

$\therefore m_1 = \frac{5}{2}$ PERPENDICULAR SLOPE: $m_2 = -\frac{2}{5}$

$\therefore$ POINT-SLOPE FORM:

$y-1 = -\frac{2}{5}(x-(-4))$

$y-1 = -\frac{2}{5}(x+4)$

SLOPE-INTERCEPT FORM:

$y = mx+b$

$1 = -\frac{2}{5}(-4)+b$

$1 = \frac{8}{5}+b$

$1-\frac{8}{5} = b$

$-\frac{3}{5} = b$

$\therefore y = -\frac{2}{5}x - \frac{3}{5}$

GRAPH AND PART b) ON NEXT PAGE

3) a) $1206\ \frac{lb}{ft^3} = 1206\ \frac{\left(\frac{1}{2.20}kg\right)}{(30.5\,cm)^3} = 0.01932\ kg/cm^3$

b) $1206\ \frac{lb}{ft^3} = 1206\ \frac{lb}{(12\,in)^3} = 0.6979\ lb/in^3$

c) $100\ \frac{km}{hr} = 100\ \frac{(10^3 m)}{hr} = 100(10)^3\ \frac{(3.28\,ft)}{(3600\,s)} = 90\ ft/s$

d) $10000\text{psi} = 10000\left(\frac{1}{1.45\times10^{-4}}\text{ Pa}\right) = 7\times10^{7}\text{Pa} = 70\text{ MPa}$

b) $x^2+y^2-6x+4y-3=0$

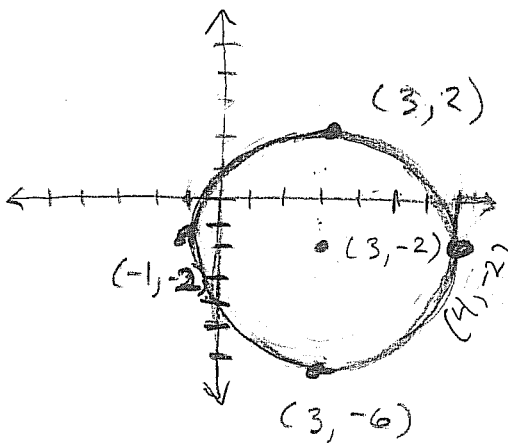
$x^2-6x+y^2+4y-3=0$

$(x^2-6x+9)-9+(y^2+4y+4)-4-3=0$

$(x-3)^2+(y+2)^2-16=0$

$(x-3)^2-(y+2)^2=4^2$

CENTRE: $(3,-2)$ RADIUS: 4



GRAPH FROM a)

x-int: $y=0$

$5x-2(0)=6$

$x=6/5$

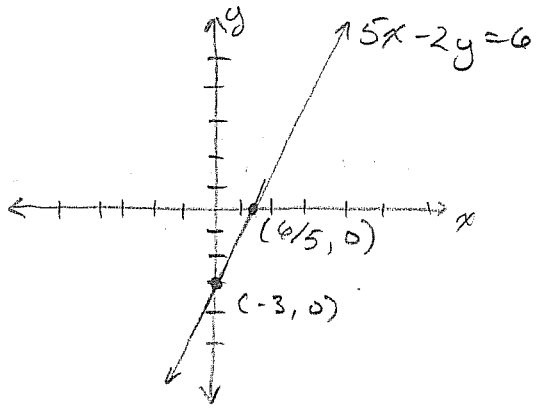
$\therefore (6/5,0)$

y-int: $x=0$

$5(0)-2y=6$

$y=-3$

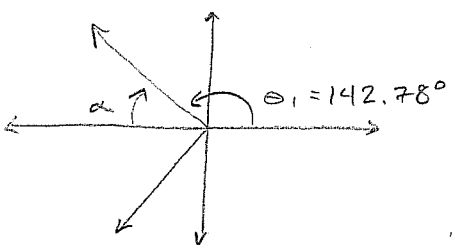
$\therefore (0,-3)$



4) a) $\csc\theta = 1.615 = \frac{1}{\sin\theta} \Rightarrow \sin\theta = \frac{1}{1.615} = 0.6192$

$\tan\theta = \tan(\sin^{-1}0.6192) = 0.7885$

b) $\theta = \cos^{-1}(-0.7963) = 142.78°$



$\cos\theta<0, \tan\theta>0$

$\therefore \theta$ IS IN QUADRANT III

$\alpha = 180° - 142.78 = 37.22°$

$\therefore \theta = 180° + 37.22° = 217.22°$

5 a) $\theta = 55.3^\circ \left(\frac{\pi}{180^\circ}\right)$: (RADIANS)

AREA OF SMALL SECTOR

$A = \frac{1}{2}\left(55.3^\circ\left(\frac{\pi}{180^\circ}\right)\right)(10.0)^2 = 48.25835\ m^2$

AREA OF LARGE SECTOR:

$A = \frac{1}{2}\left(55.3^\circ\left(\frac{\pi}{180^\circ}\right)\right)(16.0)^2 = 123.541386\ m^2$

$\therefore$ AREA OF ROAD $= 123.541386 - 48.25835 = 75.3\ m^2$

LENGTH OF GUARD RAIL:

$s = \theta r = 55.3^\circ\left(\frac{\pi}{180^\circ}\right)(16.0) = 15.4\ m$

b) NEXT PAGE.

6) $A_x = A\cos\theta_A = 432\cos 105.1^\circ = -113$

$A_y = A\sin\theta_A = 432\sin 105.1^\circ = 417$

$B_x = B\cos\theta_B = 117\cos 148.2^\circ = -99.4^\circ$

$B_y = B\sin\theta_B = 117\sin 148.2^\circ = 61.7^\circ$

LET $\vec{A} + \vec{B} = \vec{R}$

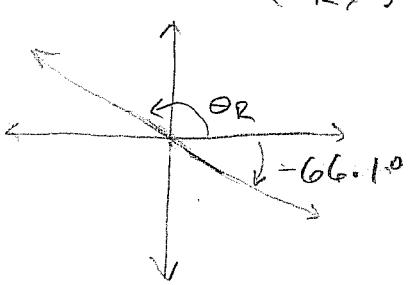
THEN $R_x = A_x + B_x$
$= -113 + (-99.4)$
$= -212$

AND $R_y = A_y + B_y = 417 + 61.7$
$= 479$

$\therefore$ MAGNITUDE: $R = \sqrt{R_x^2 + R_y^2} = \sqrt{479^2 + (-212)^2} = \sqrt{274359}$
$= 524$

DIRECTION: $\tan\theta_R = \frac{R_y}{R_x}$

$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{479}{-212}\right) = -66.1^\circ$



$\therefore \theta_R = 180^\circ - 66.1^\circ$
$= 113.9^\circ$

5b) $20.0 \frac{r}{min} = 20.0(2\pi) \frac{rad}{min}$

$v = \omega r = 20.0(2\pi) \frac{rad}{min} (2.59 m) = 325 m/min$