Dawson College: Civil Technology: 201-912-DW 02

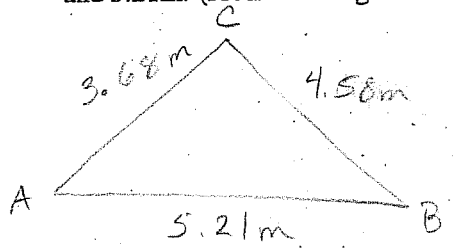
October 23, 2009

Name: SOLUTIONS

Student ID: _____

Test 2

Question 1. *(5 marks)* Three pieces of wood need to be used to build the sides of a triangular roof truss. At what angles should the pieces be joined if the pieces of wood have lengths 3.68m, 4.58m and 5.21m. (round the angles to one decimal place)



$\therefore a^2 = b^2 + c^2 - 2bc\cos A$

$4.58^2 = 3.68^2 + 5.21^2 - 2(3.68)(5.21)\cos A$

$\Rightarrow \cos A = \dfrac{4.58^2 - 3.68^2 - 5.21^2}{-2(3.68)(5.21)}$

$= 0.514012037\ldots$

$\therefore A = \cos^{-1}(0.514012037\ldots) = 59.1°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$3.68^2 = 4.58^2 + 5.21^2 - 2(4.58)(5.21)\cos B$

$\therefore \cos B = \dfrac{3.68^2 - 4.58^2 - 5.21^2}{-2(4.58)(5.21)} = 0.724549279\ldots$

$\therefore B = \cos^{-1}(0.724549279\ldots) = 43.6°$

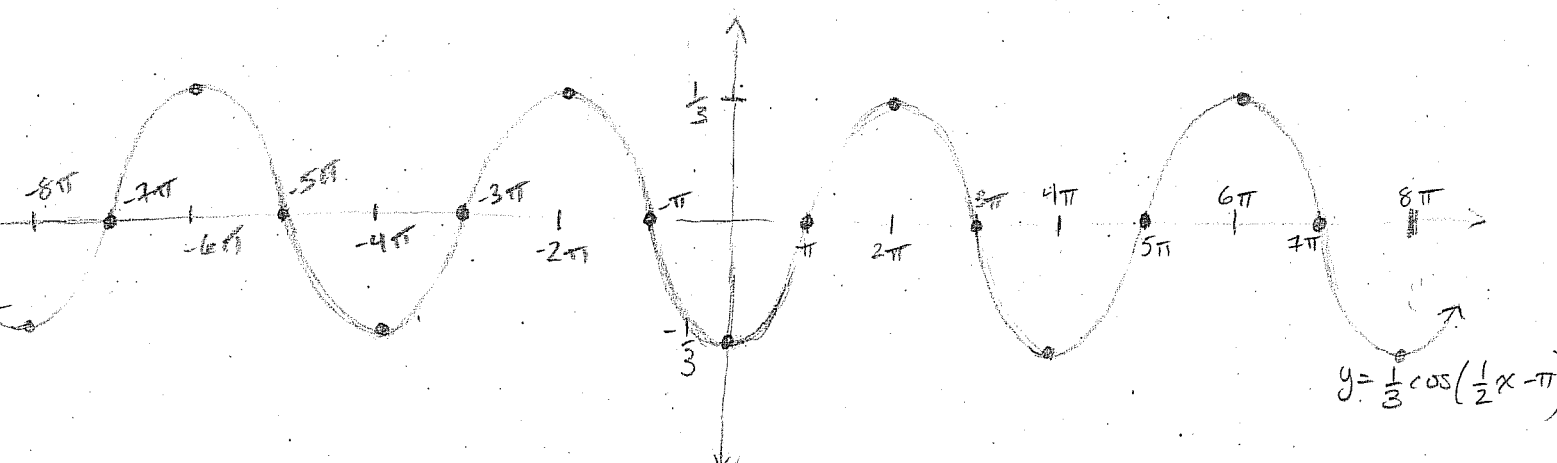
$\therefore C = 180° - 59.1° - 43.6° = 77.3°$

$\therefore$ **A = 59.1° B = 43.6° C = 77.3°**

Question 2. *(5 marks)* Find the amplitude, period, and displacement (phase shift) of $y = \frac{1}{3}\cos(\frac{1}{2}x - \pi)$. Neatly graph this function with 2 periods on either side of the y-axis (4 periods total) clearly indicating relevant points.

AMPLITUDE: $|\frac{1}{3}| = \frac{1}{3}$, PERIOD: $\frac{2\pi}{\frac{1}{2}} = 4\pi$, DISPLACEMENT: $\frac{-(-\pi)}{\frac{1}{2}} = 2\pi$

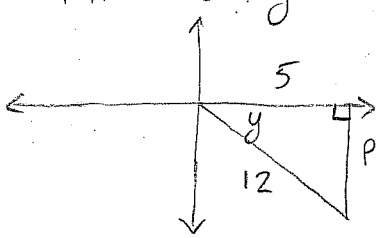
x	π	2π	3π	4π	5π
y	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0



Question 3. *(6 marks)* Given $\sin x = \frac{3}{5}$ with x in the first quadrant and $\cos y = \frac{5}{12}$ with y in the fourth quadrant find (express your answer without decimals):

(a) $\sin x + \sin y$

FIND $\sin y$:



$5^2 + p^2 = 12^2$

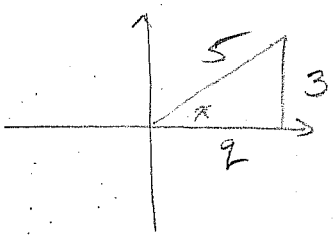
$p^2 = 12^2 - 5^2$

$= 119$

$\therefore p = -\sqrt{119} \Rightarrow \sin y = -\frac{\sqrt{119}}{12}$

$\therefore \sin x + \sin y = \frac{3}{5} - \frac{\sqrt{119}}{12} = \frac{36 - 5\sqrt{119}}{60}$

(b) $\cos(x+y) = \cos x \cos y - \sin x \sin y$ FIND $\cos x$



$3^2 + q^2 = 5^2$

$q^2 = 5^2 - 3^2$

$= 16$

$\therefore q = 4 \Rightarrow \cos x = \frac{4}{5}$

$\therefore \cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \frac{4}{5} \cdot \frac{5}{12} - \frac{3}{5} \cdot \left(\frac{-\sqrt{119}}{12}\right) = \frac{20}{60} + \frac{3\sqrt{19}}{60}$

$= \frac{20 + 3\sqrt{119}}{60}$

(c) $\tan 2y = \frac{2\tan y}{1 - \tan^2 y}$

FIND $\tan y$:

FROM a) $\tan y = -\frac{\sqrt{119}}{5}$

$\therefore \tan 2y = \frac{2\left(-\frac{\sqrt{119}}{5}\right)}{1 - \left(-\frac{\sqrt{119}}{5}\right)^2} = \frac{-2\sqrt{115}}{\frac{5}{1 - \frac{119}{25}}} = \frac{-2\sqrt{119}}{\frac{5}{\frac{25 - 119}{25}}}$

$= \frac{-2\sqrt{119}}{\frac{5}{-94}} = \frac{-2\sqrt{119}}{5} \cdot \frac{25^5}{-94} = \frac{5\sqrt{119}}{47}$

Question 4. *(5 marks)* Solve the following the following system using Cramer's rule:

$$\begin{aligned} 3x - 7y + 3z &= 6 \\ 3x + 3y + 6y &= 1 \\ 5x - 5y + z &= 5 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 6 & -7 & 3 \\ 1 & 3 & 6 \\ 5 & -5 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -7 & 3 \\ 3 & 3 & 6 \\ 5 & -5 & 1 \end{vmatrix}} = \frac{18 - 210 - 15 - 45 + 180 + 7}{9 - 210 - 45 - 45 + 90 + 21} = \frac{-65}{-180}$$

$$= \frac{13}{36}$$

$$y = \frac{\begin{vmatrix} 3 & 6 & 3 \\ 3 & 1 & 6 \\ 5 & 5 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -7 & 3 \\ 3 & 3 & 6 \\ 5 & -5 & 1 \end{vmatrix}} = \frac{3 + 180 + 45 - 15 - 90 - 18}{-180} = \frac{105}{-180}$$

$$= \frac{-7}{12}$$

$$z = \frac{\begin{vmatrix} 3 & -7 & 6 \\ 3 & 3 & 1 \\ 5 & -5 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -7 & 3 \\ 3 & 3 & 6 \\ 5 & -5 & 1 \end{vmatrix}} = \frac{45 - 35 - 90 - 90 + 15 + 105}{-180} = \frac{-50}{-180}$$

$$= \frac{5}{18}$$

$$\therefore x = \frac{13}{36} \quad y = -\frac{7}{12} \quad z = \frac{5}{18}$$

Question 5. *(5 marks)* An alloy used in electrical transformers contains nickel (Ni), Iron (Fe) end Molybdenum (Mo). The percent of Ni is 1% less than five times the percent of Fe. The percent of Fe is 1% more than three times the percent of Mo. Find the percent of each in the alloy.

$Ni + Fe + Mo = 100$

$Ni = 5Fe - 1$

$Fe = 3Mo + 1$

$\Rightarrow$

$Ni + Fe + Mo = 100$ ①

$Ni - 5Fe = -1$ ②

$Fe - 3Mo = 1$ ③

① $Ni + Fe + Mo = 100$

② $-(Ni - 5Fe = -1)$

①′ $6Fe + Mo = 101$

$3 \times$①′ : $18Fe + 3Mo = 303$

$+(Fe - 3Mo = 1)$

$19Fe = 304$

$Fe = 16$

$\therefore Ni = 5(16) - 1 = 79 \Rightarrow Mo = 100 - 79 - 16 = 5$

$\therefore$ THE ALLOY HAS 79% Ni, 5% Mo AND 16% Fe

Question 6. *(5 marks)* Find the intercepts, vertex and axis of symmetry of $y = -3x^2 - x$ and graph.

x-int: $y = 0$

$0 = -3x^2 - x$

$= -x(3x - 1)$

$\Rightarrow x = 0, \quad 3x - 1 = 0$

$x = -\frac{1}{3}$

$\therefore (0,0), (-\frac{1}{3}, 0)$

y-int: $x = 0$

$y = 0$

$\therefore (0,0)$

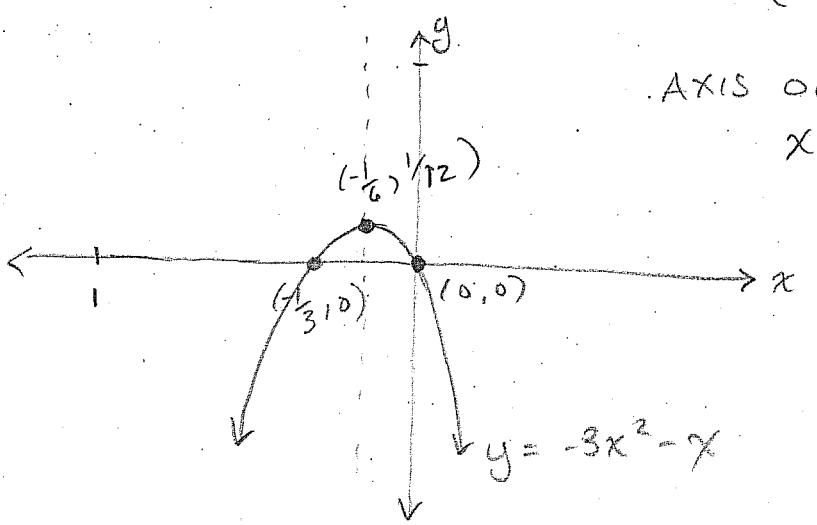
VERTEX: $x = \frac{-b}{2a}$

$x = \frac{-(-1)}{2(-3)} = -\frac{1}{6}$

$y = -3(-\frac{1}{6})^2 - (-\frac{1}{6})$

$= -\frac{3}{36} + \frac{1}{6} = \frac{3}{36} = \frac{1}{12}$

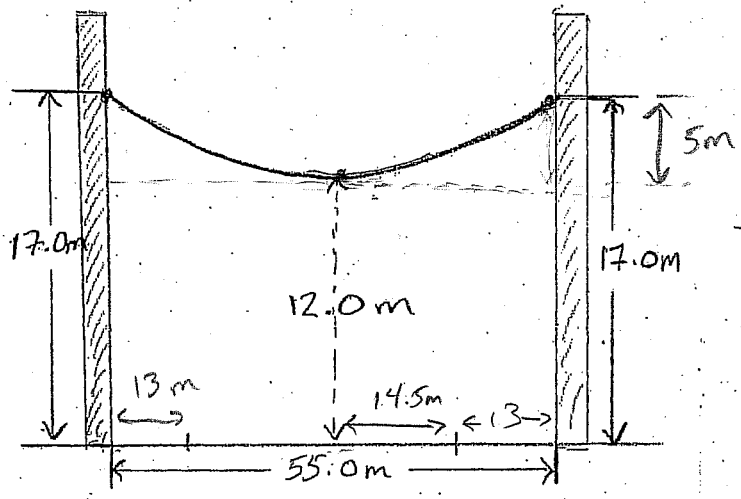
$\therefore (-\frac{1}{6}, \frac{1}{12})$

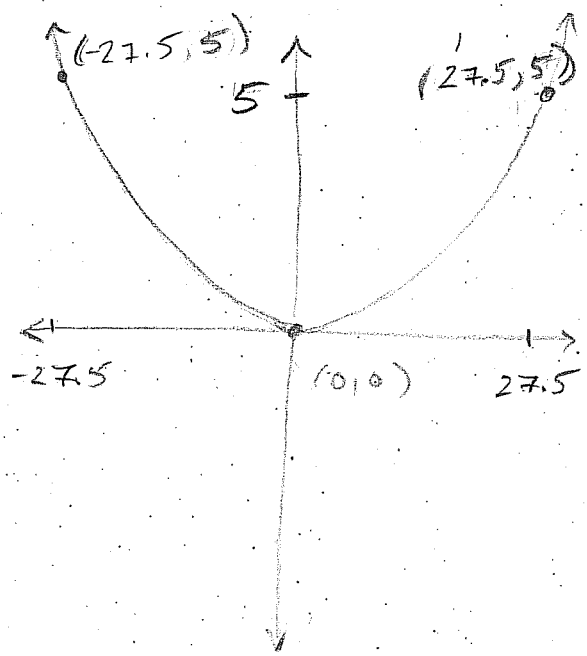


AXIS OF SYMMETRY:

$x = -\frac{1}{6}$

Question 7. *(5 marks)* A wire is fastened 17.0m up on each of two telephone poles that are 55.0m apart. Halfway between the poles the wire is 12.0m above the ground. Given that the wire is parabolic find the height of the wire 13m from either pole.





EQUATION OF PARABOLA: USE 3 POINTS: (0,0), (27.5, 5), (-27.5, 5)

$y = ax^2 + bx + c \Rightarrow 0 = a(0)^2 + b(0) + c \Rightarrow c = 0$

NOW $5 = a(27.5)^2 + b(27.5) + 0 \Rightarrow 5 = 756.25a + 27.5b$

$5 = a(27.5)^2 + b(27.5) + 0 \Rightarrow 5 = 756.25a - 27.5b$

$5 = 756.25a + 27.5b$

$-(5 = 756.25a - 27.5b)$

$0 = 55.0b$

$b = 0$

$\therefore 5 = 756.25a + 0 \Rightarrow a = 0.00661$

$\therefore y = 0.00661x^2$

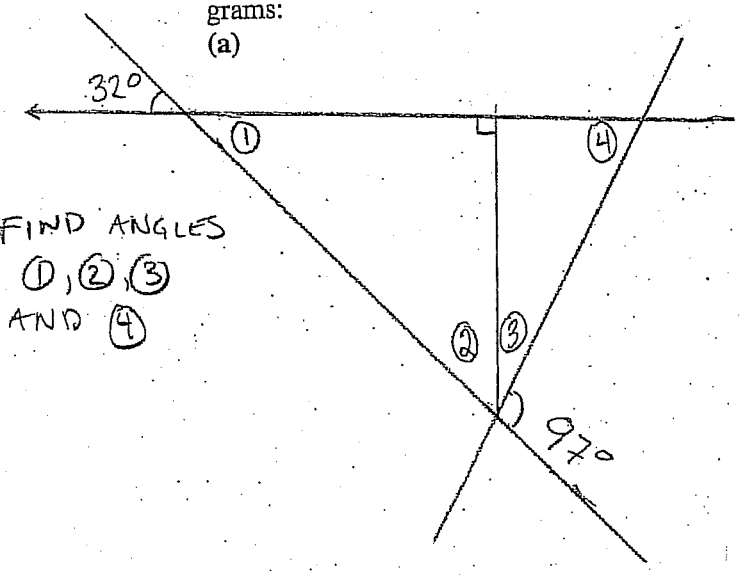
AT 13m $x = 27.5 - 13 = 14.5$m

$y = 0.00661(14.5)^2 = 1.3897525$m

$\therefore$ HEIGHT OF WIRE IS $12.0\text{m} + 1.3897525\text{m} \approx 13.4$m

Question 8. *(10 marks)* Find the numbered parts (angles or line segments) in the following diagrams:

(a)



FIND ANGLES ①, ②, ③ AND ④

$\angle 1 = 32°$

$\angle 2 = 180° - 32° - 90° = 58°$

$\angle 3 = 180° - 97° - 58°$

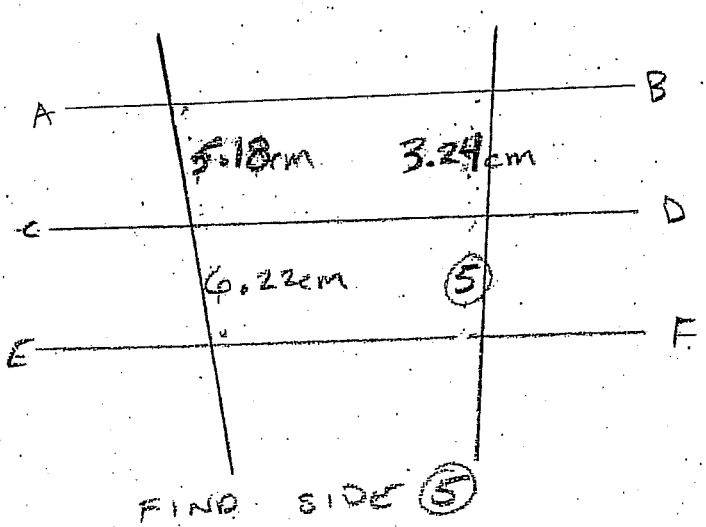
$= 25°$

$\angle 4 = 180° - 32° - (25° + 58°)$

$= 65°$

(b)

AB || CD AND CD || EF



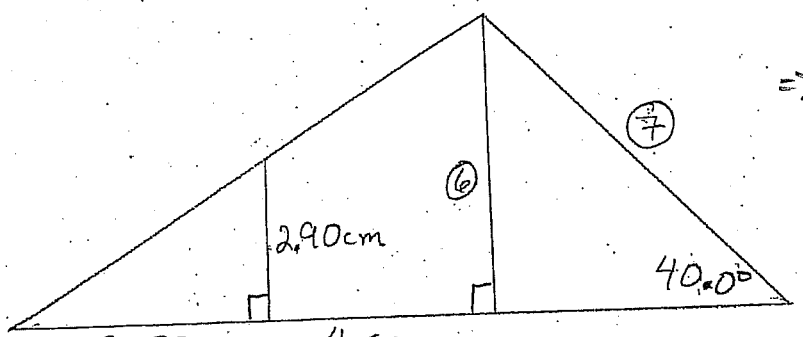
FIND SIDE ⑤

$\frac{⑤}{6.22} = \frac{3.24}{5.18}$

$⑤ = \frac{(6.22)(3.24)}{5.18}$

$= 3.89\text{cm}$

(c)



FIND SIDES ⑥, AND ⑦

$$\frac{2.90}{3.55} = \frac{⑥}{3.55+4.82}$$

$$\Rightarrow ⑥ = \frac{(8.37)(2.90)}{3.55}$$

$$= \underline{6.84\text{cm}}$$

$$\sin 40.0° = \frac{6.84}{⑦}$$

$$\therefore ⑦ = \frac{6.84}{\sin 40.0°} = \underline{10.6\text{cm}}$$

Question 9. *(5 marks)* Solve the following system:

$$① \quad 2y^2 - 4x = 7$$
$$② \quad y^2 + 2x^2 = 3$$

$$\Rightarrow \quad 2y^2 - 4x = 7$$
$$-(2y^2 + 4x^2 = 6)$$
$$-4x^2 - 4x = 1$$

$$\therefore 4x^2 + 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(4) \pm \sqrt{(4)^2-4(4)(1)}}{2(4)} = \frac{-4 \pm \sqrt{0}}{8}$$

$$= -\frac{1}{2}$$

FROM ①

$$2y^2 - 4\left(-\frac{1}{2}\right) = 7$$
$$2y^2 = 7 - 2$$
$$y^2 = \frac{5}{2} \Rightarrow y = \pm\sqrt{\frac{5}{2}}$$

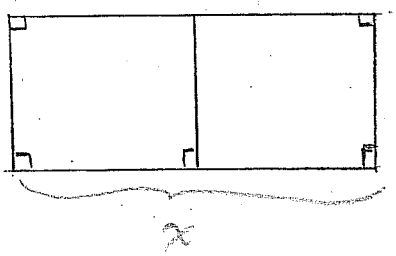
SOLUTIONS

$x = -\frac{1}{2}$ $y = \sqrt{5/2}$

OR

$x = -\frac{1}{2}$ $y = -\sqrt{5/2}$

Question 10. *(6 marks)* Security fencing encloses a rectangular storage are of $1600m^2$ that is divided into two sections by additional fencing parallel to the shorter sides. Find the dimensions of the storage area if 220m of fencing are used.



$2x + 3y = 220$

$xy = 1600 \Rightarrow y = \frac{1600}{x}$

FROM (1) $2x + 3\left(\frac{1600}{x}\right) = 220$

$2x + \frac{4800}{x} = 220$

$2x^2 + 4800 = 220x$

$2x^2 - 220x + 4800 = 0$

$x^2 - 110x + 2400 = 0$

$(x-80)(x-30) = 0$

$\therefore x = 80, 30$

FROM (2)

$x = 30 \Rightarrow y = \frac{1600}{30} = 53.3$

$x = 80 \Rightarrow y = \frac{1600}{80} = 20$

$\therefore$ THE DIMENSIONS ARE

80m BY 20m OR 53.3m BY 30m

Bonus *(2 marks)* Solve for x.

$$\begin{vmatrix} 3 & 0 & -1 \\ 1 & x & 2 \\ 1 & 2 & x+1 \end{vmatrix} = \begin{vmatrix} 2 & x \\ -1 & x+2 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 0 & -1 \\ 1 & x & 2 \\ 1 & 2 & x+1 \end{vmatrix} \begin{matrix} 3 & 0 \\ 1 & x \\ 1 & 2 \end{matrix} = 3(x)(x+1) + 0 - 2 + x - 12 - 0$$

$$= 3x^2 + 3x - 2 + x - 12$$

$$= 3x^2 + 4x - 14$$

$$\begin{vmatrix} 2 & x \\ -1 & x+2 \end{vmatrix} = 2(x+2) - (x)(-1) = 2x + 4 - x = 3x + 4$$

$\therefore 3x^2 + 4x - 14 = 3x + 4 \Rightarrow 3x^2 + x - 18 = 0$

$\therefore x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-18)}}{2(3)} = \frac{-1 \pm \sqrt{217}}{6}$