Dawson College: Civil Technology: 201-912-DW 02

November 20, 2009

Last Name: SOLUTIONS

First Name:

Student ID:

Test 3

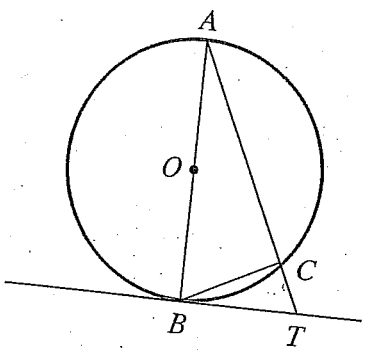
Question 1. *(5 marks)* In the following diagram AB is a diameter, TB is a tangent line at B and $\angle ABC = 65°$. Find the following angles:

(a) $\angle CBT$

(b) $\angle BCT$

(c) $\angle CAB$

(d) $\angle BTC$



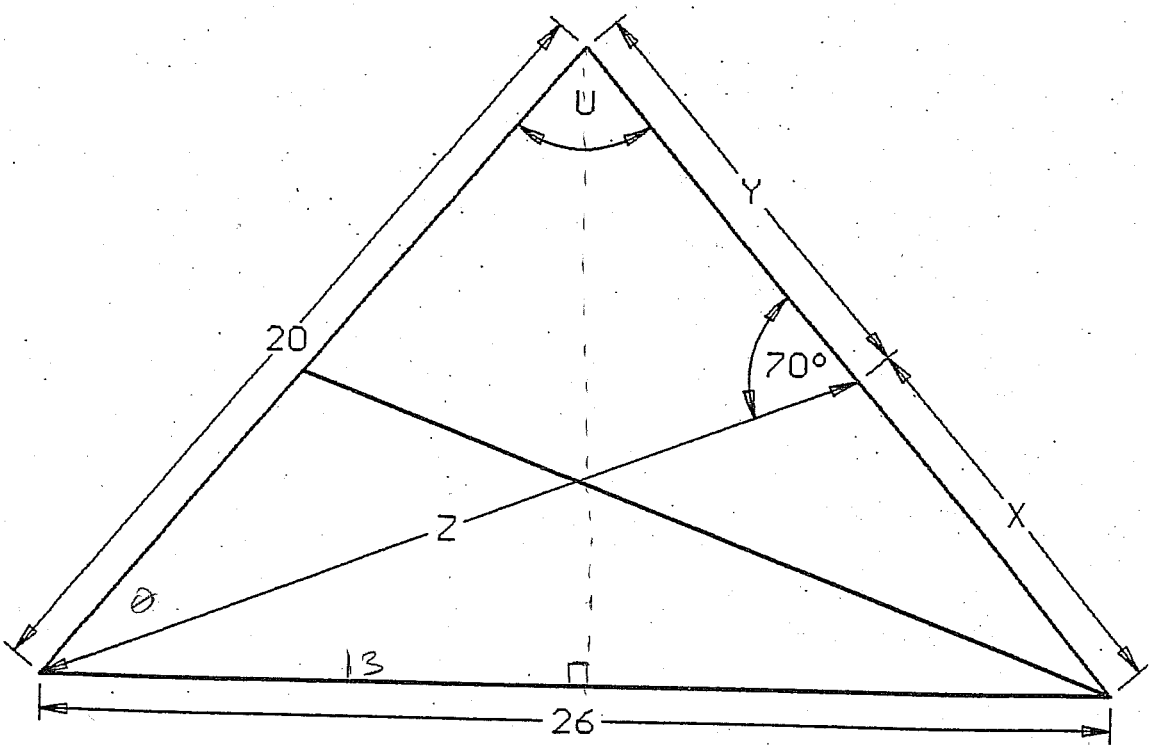
a) $\angle CBT = 90 - 65 = 25°$

b) $\angle BCT = 180 - \angle ACB = 180 - 90 = 90°$

c) $\angle CAB = 180° - 90° - 65° = 25°$

d) $\angle BTC = 180° - 90° - 25° = 65°$

Question 2. *(10 marks)* The following is a diagram of a symmetric scissortruss. Find X, Y, Z, and U. (m)



$\sin\left(\frac{1}{2}U\right) = \frac{13}{20} \Rightarrow \frac{1}{2}U = \sin^{-1}\left(\frac{13}{20}\right) = 40.54160187°$

$\therefore U = 81.08320375°$

$\frac{20}{\sin 70°} = \frac{Z}{\sin U} \Rightarrow Z = \frac{20(\sin 81.08320375°)}{\sin 70°} = 21.02633244\,m$

$\theta = 180 - 70° - U = 28.91679625°$

$\frac{Y}{\sin\theta} = \frac{20}{\sin 70°} \Rightarrow Y = \frac{20\sin\theta}{\sin 70°} = 10.29142922\,m$

$X = 20 - Y = 20 - 10.29142922 = 9.708570779\,m$

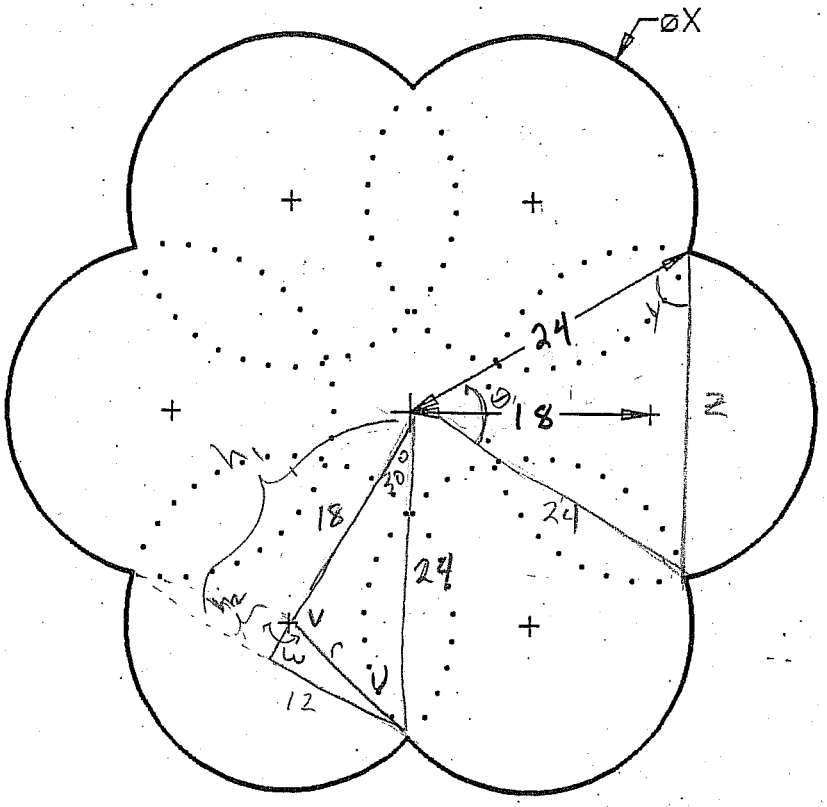
$$= 6\left[\frac{1}{2}(24)(20.784\,60969)\right.$$
$$+ \frac{1}{2}\left(153.871\,3731^{\circ}\frac{\pi}{180^{\circ}}\right)(12.318\,84942)^2$$
$$\left. - \frac{1}{2}(24)(2.784\,60969)\right]$$

$$= 6\,(249.415\,3163 + 203.772\,4629 - 33.415\,31628)$$

$$= 6\,(419.772\,4629) = \underline{2518.634\,778\,m^3}$$

VOLUME $= (2518.634\,778)(0.35) = 881.521\,723\,m^3$

Question 3. *(10 marks)* The following is a floorplan. Find X the circumference, area and volume given that the thickness is 0.35m. All lengths are in metres.



$$\theta = \frac{360^\circ}{6} = 60^\circ$$

$2Y = 180 - 60^\circ = 120^\circ$

$Y = 60^\circ$

$\therefore Z = 24$ (ISOSCELES TRIANGLE)

$r^2 = 18^2 + 24^2 - 2(18)(24)\cos 30^\circ$

$r = 12.318849 42$

$\therefore X = 2r = \underline{24.637698 85 m}$

$24^2 = 18^2 + r^2 - 2(18)r\cos V \Rightarrow \cos V = \frac{24^2 - 18^2 - r^2}{-2(18)r} = -0.321711743$

$\therefore V = \cos^{-1}(-0.321711743)$

$= 103.064313 40^\circ$

$W = 360^\circ - 2V = 153.871373 1^\circ$

CIRCUMFERENCE $= 6 \cdot (\theta r)$

$= 6\left(153.8713731 \cdot \frac{\pi}{180^\circ}\right)(12.318849 42) = 198.4982096 m$

$h_1^2 = \sqrt{24^2 - 12^2} = 20.784609 69$

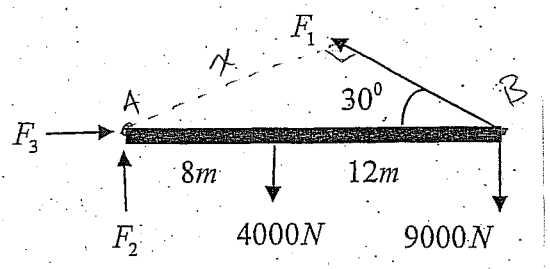
$h_2 = 20.784609 69 - 18$

$= 2.784609 69$

$\therefore$ AREA $= 6\Big($(AREA OF LARGE TRIANGLE) + (AREA OF SECTOR) − (AREA OF SMALL TRIANGLE)$\Big)$

$=$

Question 4. *(7 marks)* A 20m crane arm with a supporting cable F_1 is carrying a suspended load of 9000N. Other forces are shown. Given that the following is in equilibrium find forces F_1, F_2 and F_3:



@A $\sin 30° = \frac{x}{20}$

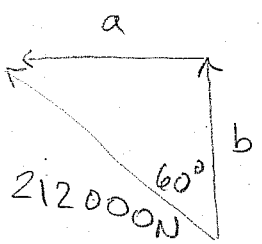
$\Rightarrow x = 20 \sin 30° = 10$

$\overrightarrow{OM} = \overrightarrow{OM}$

$8(4000) + 20(9000) = (20 \sin 30°) F_1$

$\therefore F_1 = \frac{32\,000 + 180\,000}{10}$

$= 21200N$



$a = 21200 \cdot \sin 60° = 18359.73856N$

$b = 21200 \cdot \cos 60° = 10600N$

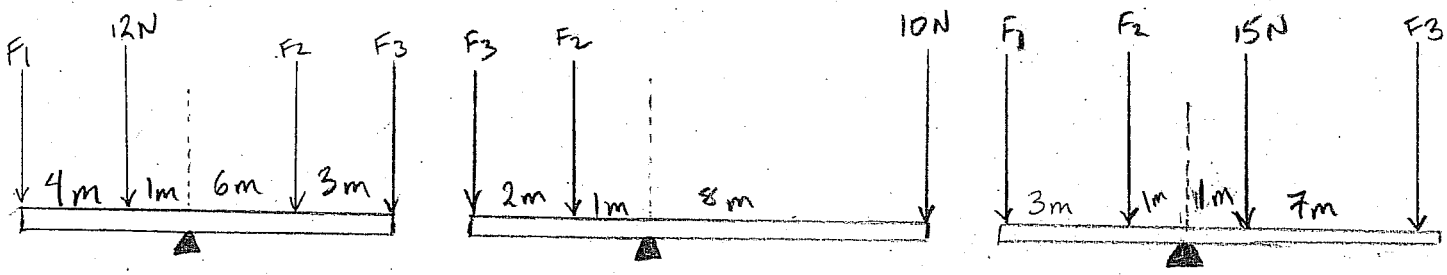
$\therefore F_3 = a = 18400N$

$F_2 + b = 4000 + 9000$

$F_2 = 13000 - 10600$

$= 2400N$

Question 5. *(10 marks)* The following three systems are all in equilibrium with the same forces F_1, F_2 and F_3. Find these forces.



① $5F_1 + (1)(12) = 6F_2 + 9F_3 \Rightarrow 5F_1 - 6F_2 - 9F_3 = -12$

② $3F_3 + F_2 = 8(10) \Rightarrow F_2 + 3F_3 = 80$

③ $4F_1 + F_2 = (1)(15) + 8F_3 \Rightarrow 4F_1 + F_2 - 8F_3 = 15$

① × 4: $20F_1 - 24F_2 - 36F_3 = -48$

③ × 5: $20F_1 + 5F_2 - 40F_3 = 75$

$-29F_2 + 4F_3 = -123$

② × 29: $29F_2 + 87F_3 = 2320$

$91F_3 = 2197$

$F_3 = 24.1\text{N}$

$\therefore F_2 + 3(24.1) = 80$

$F_2 = 80 - 72.3$

$F_2 = 7.70\text{N}$

$5F_1 - 6(7.70) - 9(24.1) = -12$

$F_1 = 50.1\text{N}$

Question 6. *(10 marks)* Find the point(s) of intersection of the line passing through the point $(0,-625/24)$ and parallel to the line $24y=7x-14$ and the circle $x^2+y^2=625$. Decide whether this line is a tangent line, secant line, or neither. (do not use decimals)

$24y = 7x - 14$

$y = \frac{7}{24}x - \frac{14}{24} \quad \therefore \quad m = \frac{7}{24}$ PARALLEL

$y = mx + b$

$\frac{-625}{24} = \frac{7}{24}(0) + b \Rightarrow b = \frac{-625}{24} \Rightarrow y = \frac{7}{24}x - \frac{625}{24}$

POINT(S) OF INTERSECTION:

$x^2 + y^2 = 625$

$x^2 + \left(\frac{7}{24}x - \frac{625}{24}\right)^2 = 625$

$x^2 + \frac{49}{576}x^2 - \frac{4375x}{576} - \frac{4375}{576}x + \frac{390625}{576} = 625$

$576x^2 + 49x^2 - 4375x - 4375x + 390625 = 360000$

$625x^2 - 8750x + 30625 = 0$

$625(x^2 - 14x + 49) = 0$

$x^2 - 14x + 49 = 0$

$(x-7)(x-7) = 0$

$x = 7$

FIND y: $y = \frac{7}{24}(7) - 625 = \frac{49}{24} - \frac{625}{24} = -\frac{576}{24}$

$= -24$

$\therefore$ THE POINT OF INTERSECTION IS $(7, -24)$

$\therefore$ IT IS A TANGENT LINE.

Question 7. *(8 marks)* Isolate for y, simplify as much as possible.

(a) $\log_7 y = -2\log_7(x+1) + \log_7 4$

$$\log_7 y = \log_7 (x+1)^{-2} + \log_7 4$$

$$\log_7 y = \log_7 \frac{4}{(x+1)^2}$$

$$y = \frac{4}{(x+1)^2}$$

(b) $\log_3 y = 5\log_3 x + \frac{\log_3 4}{2}$

$$\log_3 y = \log_3 x^5 + \log_3 4^{1/2}$$

$$\log_3 y = \log_3 2x^5$$

$$y = 2x^5$$

(c) $\log_4 y + 5\log_4 x = 1$

$$\log_4 y + \log_4 x^5 = \log_4 4$$

$$\log_4 yx^5 = \log_4 4$$

$$yx^5 = 4$$

$$y = \frac{4}{x^5}$$

Question 8. *(5 marks)* Express each as a sum, difference, or multiple of logarithms (separate). Show all steps.:

(a) $\log_2\left(\frac{xy^2}{z^3}\right)$

$= \log_2 xy^2 - \log_2 z^3$

$= \log_2 x + \log_2 y^2 - 3\log_2 z$

$= \log_2 x + 2\log_2 y - 3\log_2 z$

(b) $\log_9\left(\frac{x}{\sqrt{yz}}\right)$

$= \log_9 x - \log_9 (xy)^{1/2}$

$= \log_9 x - \frac{1}{2}\log_9 xy$

$= \log_9 x - \frac{1}{2}\log_9 x - \frac{1}{2}\log_9 y$