EXAMPLE 11

$$\frac{1}{x^{-1}}\left(\frac{\overbrace{x^{-1} - y^{-1}}^{\text{terms}}}{x^2 - y^2}\right) = \frac{x}{1}\left(\frac{\frac{1}{x} - \frac{1}{y}}{x^2 - y^2}\right) = x\left(\frac{\frac{y-x}{xy}}{x^2 - y^2}\right)$$

$$= \frac{\frac{x(y-x)}{xy}}{(x-y)(x+y)}$$

$$= \frac{x(y-x)}{xy} \times \frac{1}{(x-y)(x+y)}$$

$$= \frac{x(y-x)}{xy(x-y)(x+y)} = \frac{-(x-y)}{y(x-y)(x+y)}$$

$$= -\frac{1}{y(x+y)}$$

CAUTION Note that in this example ***the x^{-1} and y^{-1} in the numerator could not be moved directly to the denominator with positive exponents*** because they are only terms of the original numerator.

EXAMPLE 12 $3(x+4)^2(x-3)^{-2} - 2(x-3)^{-3}(x+4)^3$

$$= \frac{3(x+4)^2}{(x-3)^2} - \frac{2(x+4)^3}{(x-3)^3} = \frac{3(x-3)(x+4)^2 - 2(x+4)^3}{(x-3)^3}$$

$$= \frac{(x+4)^2[3(x-3) - 2(x+4)]}{(x-3)^3}$$

$$= \frac{(x+4)^2(x-17)}{(x-3)^3}$$

Expressions such as the one in this example are commonly found in problems in calculus.

EXERCISES 11.1

In Exercises 1–4, solve the resulting problems if the given changes are made in the indicated examples of this section.

1. In Example 4, change the factor x^2 to x^{-2} and then find the result.
2. In Example 7, change term $2a$ to $2a^{-1}$ and then find the result.
3. In Example 10, change the 3^{-1} in the denominator to 3^{-2} and then find the result.
4. In Example 11, change the sign in the numerator from $-$ to $+$ and then find the result.

In Exercises 5–52, express each of the given expressions in simplest form with only positive exponents.

5. x^7x^{-4}
6. y^9y^{-2}
7. $2a^2a^{-6}$
8. $5ss^{-5}$
9. 5×5^{-3}
10. $(3^2 \times 4^{-3})^3$
11. $(2\pi x^{-1})^2$
12. $(3xy^{-2})^3$
13. $(5an^{-2})^{-1}$
14. $(6s^2t^{-1})^{-2}$
15. $(-4)^0$
16. -4^0
17. $-7x^0$
18. $(-7x)^0$
19. $3x^{-2}$
20. $(3x)^{-2}$
21. $(7a^{-1}x)^{-3}$
22. $7a^{-1}x^{-3}$
23. $\left(\frac{2}{n^3}\right)^{-3}$
24. $\left(\frac{3}{x^3}\right)^{-2}$
25. $\left(\frac{a}{b^{-2}}\right)^{-3}$
26. $\left(\frac{2n^{-2}}{D^{-1}}\right)^{-2}$
27. $(a+b)^{-1}$
28. $a^{-1} + b^{-1}$
29. $3x^{-2} + 2y^{-2}$
30. $(3x + 2y)^{-2}$
31. $(2a^{-n})^2\left(\frac{3}{2a^n}\right)^{-1}$
32. $(7 \times 3^{-a})\left(\frac{3^a}{7}\right)^2$

33. $\left(\frac{3a^2}{4b}\right)^{-3}\left(\frac{4}{a}\right)^{-5}$

34. $(2np^{-2})^{-2}(4^{-1}p^2)^{-1}$

35. $\left(\frac{V^{-1}}{2t}\right)^{-2}\left(\frac{t^2}{V^{-2}}\right)^{-3}$

36. $\left(\frac{a^{-2}}{b^2}\right)^{-3}\left(\frac{a^{-3}}{b^5}\right)^2$

37. $2a^{-2} + (2a^{-2})^4$

38. $3(a^{-1}z^2)^{-3} + c^{-2}z^{-1}$

39. $2 \times 3^{-1} + 4 \times 3^{-2}$

40. $5 \times 2^{-2} - 3^{-1} \times 2^3$

41. $(R_1^{-1} + R_2^{-1})^{-1}$

42. $(2a - b^{-2})^{-1}$

43. $(n^{-2} - 2n^{-1})^2$

44. $(2^{-3} - 4^{-1})^{-2}$

45. $\frac{6^{-1}}{4^{-2} + 2}$

46. $\frac{x - y^{-1}}{x^{-1} - y}$

47. $\frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}}$

48. $\frac{ax^{-2} + a^{-2}x}{a^{-1} + x^{-1}}$

49. $2t^{-2} + t^{-1}(t + 1)$

50. $3x^{-1} - x^{-3}(y + 2)$

51. $(D - 1)^{-1} + (D + 1)^{-1}$

52. $4(2x - 1)(x + 2)^{-1} - (2x - 1)^2(x + 2)^{-2}$

In Exercises 53–68, perform the indicated operations.

53. Express $4^2 \times 64$ (a) as a power of 4 and (b) as a power of 2.

54. Express $1/81$ (a) as a power of 9 and (b) as a power of 3.

55. (a) By use of Eqs. (11.4) and (11.6), show that

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

(b) Verify the equation in part (a) by evaluating each side with $a = 3.576$, $b = 8.091$, and $n = 7$.

W 56. For what integral values of n is $(-3)^{-n} = -3^{-n}$? Explain.

57. For what integral value(s) of n is $n^{\pi} > \pi^{n}$?

W 58. Evaluate $(8^{19})^{12}/(8^{16})^{14}$. What happens when you try to evaluate this on a calculator?

59. Solve for x: $2^{5x} = 2^7(2^{2x})^2$.

60. In analyzing the tuning of an electronic circuit, the expression $[\omega\omega_0^{-1} - \omega_0\omega^{-1}]^2$ is used. Expand and simplify this expression.

61. The metric unit of energy, the *joule* (J), can be expressed as $kg \cdot s^{-2} \cdot m^2$. Simplify these units and include *newtons* (see Appendix B) and only positive exponents in the final result.

62. The units for the electric quantity called *permittivity* are $C^2 \cdot N^{-1} \cdot m^{-2}$. Given that $1\ F = 1\ C^2 \cdot J^{-1}$, show that the units of permittivity are F/m. See Appendix B.

63. When studying a solar energy system, the units encountered are $kg \cdot s^{-1}(m \cdot s^{-2})^2$. Simplify these units and include *joules* (see Example 5) and only positive exponents in the final result.

64. The metric units for the velocity v of an object are $m \cdot s^{-1}$, and the units for the acceleration a of the object are $m \cdot s^{-2}$. What are the units for v/a?

65. Given that $v = a^p t^r$, where v is the velocity of an object, a is its acceleration, and t is the time, use the metric units given in Exercise 64 to show that $p = r = 1$.

66. An expression encountered in finance is

$$\frac{p(1 + i)^{-1}[(1 + i)^{-n} - 1]}{(1 + i)^{-1} - 1}$$

where n is an integer. Simplify this expression.

67. An idealized model of the thermodynamic process in a gasoline engine is the *Otto cycle*. The efficiency e of the process is

$$e = \frac{\frac{T_1 r^{\gamma}}{r} - \frac{T_2 r^{\gamma}}{r} - T_1 + T_2}{\frac{T_1 r^{\gamma}}{r} - \frac{T_2 r^{\gamma}}{r}}$$

Show that $e = 1 - \frac{1}{r^{\gamma - 1}}$.

68. In optics, the combined focal length F of two lenses is given by $F = [f_1^{-1} + f_2^{-1} + d(f_1 f_2)^{-1}]^{-1}$, where f_1 and f_2 are the focal lengths of the lenses and d is the distance between them. Simplify the right side of this equation.

11.2 FRACTIONAL EXPONENTS

In Section 11.1, we reviewed the use of integral exponents, including exponents that are negative integers and zero. We now show how rational numbers may be used as exponents. With the appropriate definitions, all the laws of exponents are valid for all rational numbers as exponents.

Equation (11.3) states that $(a^m)^n = a^{mn}$. If we were to let $m = \frac{1}{2}$ and $n = 2$, we would have $(a^{1/2})^2 = a^1$. However, we already have a way of writing a quantity that, when squared, equals a. This is written as $\sqrt{a}$. To be consistent with previous definitions and to allow the laws of exponents to hold, we define

$$\boxed{a^{1/n} = \sqrt[n]{a}} \qquad (11.7)$$

The radical in Eq. (11.7) is the symbol for the nth root of a number. Do not confuse it with $\sqrt{a}$, the square root of a.

EXAMPLE 9 $(4x^4)^{-1/2} - 3x^{-3} = \dfrac{1}{(4x^4)^{1/2}} - \dfrac{3}{x^3}$ using Eq. (11.6)

$$= \frac{1}{2x^2} - \frac{3}{x^3} \quad \text{using Eq. (11.7)}$$

$$= \frac{x-6}{2x^3} \quad \text{common denominator}$$

...he chapter introduction.

EXAMPLE 10 The rate R at which solar radiation changes at a solar-energy collector during a day is given by the equation

$$R = \frac{(t^4+100)^{1/2} - 2t^3(t+6)(t^4+100)^{-1/2}}{[(t^4+100)^{1/2}]^2}$$

...ar collectors supply the power for most ...ace satellites.

Here, R is measured in $\text{kW}/(\text{m}^2 \cdot \text{h})$, t is the number of hours from noon, and $-6\ \text{h} \le t \le 8\ \text{h}$. Express the right side of this equation in simpler form and find R for $t = 0$ (noon) and for $t = 4$ h (4 P.M.)

Performing the simplification, we have the following steps:

$$R = \frac{(t^4+100)^{1/2} - \dfrac{2t^3(t+6)}{(t^4+100)^{1/2}}}{(t^4+100)} \quad \begin{array}{l}\longleftarrow \text{using Eq. (11.6)} \\ \longleftarrow \text{using Eq. (11.3)}\end{array}$$

$$= \frac{\dfrac{(t^4+100)^{1/2}(t^4+100)^{1/2} - 2t^3(t+6)}{(t^4+100)^{1/2}}}{(t^4+100)} \quad \longleftarrow \text{common denominator}$$

$$= \frac{(t^4+100) - 2t^3(t+6)}{(t^4+100)^{1/2}} \times \frac{1}{t^4+100} \quad \longleftarrow \text{invert divisor and multiply}$$

$$= \frac{100 - 12t^3 - t^4}{(t^4+100)^{1/2}(t^4+100)} = \frac{100 - 12t^3 - t^4}{(t^4+100)^{3/2}} \quad \longleftarrow \text{using Eq. (11.1)}$$

$$\text{For } t = 0: \quad R = \frac{100 - 12(0^3) - 0^4}{(0^4+100)^{3/2}} = 0.10\ \text{kW}/(\text{m}^2 \cdot \text{h})$$

$$\text{For } t = 4\ \text{h}: \quad R = \frac{100 - 12(4^3) - 4^4}{(4^4+100)^{3/2}} = -0.14\ \text{kW}/(\text{m}^2 \cdot \text{h})$$

We see that the radiation is increasing at noon, and the negative sign tells us that it is decreasing at 4 P.M.

EXERCISES 11.2

...xercises 1–4, solve the resulting problems if the given changes are ...e in the indicated examples of this section.

1. In Example 2, change the exponent to 4/3 and then find the result.
2. In Example 4(b), change the exponent to $-3/2$ and then find the result.
3. In Example 6, change the exponent to $-1/3$ and then plot the graph.
4. In Example 9, change the exponent $-1/2$ to $-3/2$ and then find the result.

In Exercises 5–28, evaluate the given expressions.

5. $25^{1/2}$	**6.** $27^{1/3}$	**7.** $81^{1/4}$
8. $125^{2/3}$	**9.** $100^{25/2}$	**10.** $16^{5/4}$
11. $8^{-1/3}$	**12.** $16^{-1/4}$	**13.** $64^{-2/3}$
14. $32^{-4/5}$	**15.** $5^{1/2}5^{3/2}$	**16.** $(4^4)^{3/2}$
17. $(3^6)^{2/3}$	**18.** $\dfrac{121^{-1/2}}{100^{1/2}}$	**19.** $\dfrac{1000^{1/3}}{400^{-1/2}}$
20. $\dfrac{7^{-1/2}}{6^{-1}7^{1/2}}$	**21.** $\dfrac{15^{2/3}}{5^2 15^{-1/3}}$	**22.** $\dfrac{(-27)^{1/3}}{6}$

23. $\dfrac{(-8)^{2/3}}{-2}$ 24. $\dfrac{-4}{(-64)^{-2/3}}$

25. $125^{-2/3} - 100^{-3/2}$ 26. $32^{0.4} + 25^{-0.5}$

27. $\dfrac{16^{-0.25}}{5} + \dfrac{2^{-0.6}}{2^{0.4}}$ 28. $\dfrac{4^{-1}}{36^{-1/2}} - \dfrac{5^{-1/2}}{5^{1/2}}$

In Exercises 29–32, use a calculator to evaluate each expression.

29. $17.98^{1/4}$ 30. $750.81^{2/3}$

31. $4.0187^{-4/9}$ 32. $0.1863^{-1/6}$

In Exercises 33–56, simplify the given expressions. Express all answers with positive exponents.

33. $B^{2/3}B^{1/2}$ 34. $x^{5/6}x^{-1/3}$ 35. $\dfrac{y^{-1/2}}{y^{2/5}}$

36. $\dfrac{s^{1/4}s^{2/3}}{s^{-1}}$ 37. $\dfrac{x^{3/10}}{x^{-1/5}x^{2}}$ 38. $\dfrac{R^{-2/5}R^{2}}{R^{-3/10}}$

39. $(8a^3b^6)^{1/3}$ 40. $(8b^{-4}c^2)^{2/3}$ 41. $(16a^4b^3)^{-3/4}$

42. $(32C^5D^4)^{-2/5}$ 43. $\left(\dfrac{a^{5/7}}{a^{2/3}}\right)^{7/4}$ 44. $\left(\dfrac{4a^{5/6}b^{-1/5}}{a^{2/3}b^{2}}\right)^{-1/2}$

45. $\dfrac{1}{2}(4x^2 + 1)^{-1/2}(8x)$ 46. $\dfrac{2}{3}(x^3 + 1)^{-1/3}(3x^2)$

47. $\left(\dfrac{6x^{-1/2}y^{2/3}}{18x^{-1}}\right)\left(\dfrac{2y^{1/4}}{x^{1/3}}\right)$ 48. $\dfrac{3^{-1}a^{1/2}}{4^{-1/2}b} \div \dfrac{9^{1/2}a^{-1/3}}{2b^{-1/4}}$

49. $(T^{-1} + 2T^{-2})^{-1/2}$ 50. $(a^{-2} - a^{-4})^{-1/4}$

51. $(a^3)^{-4/3} + a^{-2}$ 52. $(4N^6)^{-1/2} - 2N^{-1}$

53. $[(a^{1/2} - a^{-1/2})^2 + 4]^{1/2}$ 54. $4x^{1/2} + \frac{1}{2}x^{-1/2}(4x + 1)$

55. $x^2(2x - 1)^{-1/2} + 2x(2x - 1)^{1/2}$

56. $(3n - 1)^{-2/3}(1 - n) - (3n - 1)^{1/3}$

In Exercises 57–60, graph the given functions.

57. $f(x) = 3x^{1/2}$ 58. $f(x) = 2x^{2/3}$

59. $f(t) = t^{-4/5}$ 60. $f(V) = 4V^{3/2}$

In Exercises 61–68, perform the indicated operations.

61. Express with fractional exponents: (a) $\sqrt[5]{x}$ and (b) $\sqrt[7]{T^3}$.

W 62. (a) Simplify $(x^2 - 4x + 4)^{1/2}$. (b) For what values of x is your answer in part (a) valid? Explain.

W 63. A factor used in determining the performance of a solar-energy storage system is $(A/S)^{-1/4}$, where A is the actual storage capacity and S is a standard storage capacity. If this factor is 0.5, explain how to find the ratio A/S.

64. A factor used in measuring the loudness sensed by the human ear is $(I/I_0)^{0.3}$, where I is the intensity of the sound and I_0 is a reference intensity. Evaluate this factor for $I = 3.2 \times 10^{-6}\ \mathrm{W/m^2}$ (ordinary conversation) and $I_0 = 10^{-12}\ \mathrm{W/m^2}$.

65. The period T of a satellite circling earth is given by $T^2 = kR^3\left(1 + \dfrac{d}{R}\right)^3$, where R is the radius of earth, d is the distance of the satellite above earth, and k is a constant. Solve for R using fractional exponents in the result.

66. The withdrawal resistance R of a nail of diameter d indicates its holding power. One formula for R is $R = ks^{5/2}dh$, where k is a constant, s is the specific gravity of the wood, and h is the depth of the nail in the wood. Solve for s using fractional exponents in the result.

67. The electric current i (in A) in a circuit with a battery of voltage E, a resistance R, and an inductance L, is $i = \dfrac{E}{R}(1 - e^{-Rt/L})$, where t is the time after the circuit is closed. See Fig. 11.3. Find i for $E = 6.20$ V, $R = 1.20\ \Omega$, $L = 3.24$ H, and $t = 0.001\,00$ s. (The number e is irrational and can be found from the calculator by using the $\boxed{e^x}$ key with $x = 1$.)

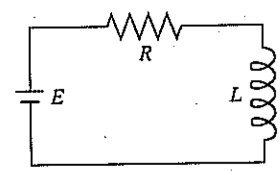

Fig. 11.3

68. For the heat-seeking rocket in pursuit of an aircraft, the distance d (in km) from the rocket to the aircraft is $d = \dfrac{500(\sin\theta)^{1/2}}{(1 - \cos\theta)^{3/2}}$, where θ is shown in Fig. 11.4. Find d for $\theta = 125.0°$.

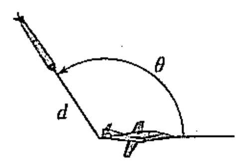

Fig. 11.4

EXERCISES 1.10

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 3, change 12 to -12 in each of the four illustrations and then solve.
2. In Example 5, change $n - 6$ to $6 - n$ on the right side and then solve.
3. In Example 6, change $(6x - 8)$ to $(8 - 6x)$ and then solve.
4. In Example 9, for the same given values of I and V, find I for $V = 48.0$ V.

In Exercises 5–40, solve the given equations.

5. $x - 2 = 7$
6. $x - 4 = 1$
7. $x + 5 = 4$
8. $s + 6 = -3$
9. $\frac{t}{2} = 5$
10. $\frac{x}{4} = -2$
11. $4E = -20$
12. $2x = 12$
13. $3t + 5 = -4$
14. $5D - 2 = 13$
15. $5 - 2y = 3$
16. $8 - 5t = 18$
17. $3x + 7 = x$
18. $6 + 4L = 5 - 3L$
19. $2(s - 4) = s$
20. $3(n - 2) = -n$
21. $6 - (r - 4) = 2r$
22. $5 - (x + 2) = 5x$
23. $2(x - 3) - 5x = 7$
24. $4(F + 7) = -7$
25. $0.1x - 0.5(x - 2) = 2$
26. $1.5x - 0.3(x - 4) = 6$
27. $7 - 3(1 - 2p) = 4 + 2p$
28. $3 - 6(2 - 3t) = t - 5$
29. $\frac{4x - 2(x - 4)}{3} = 8$
30. $2x = \frac{3 - 5(7 - 3x)}{4}$
31. $|x| - 1 = 8$
32. $2 - |x| = 4$

In Exercises 33–40, all numbers are approximate.

33. $5.8 - 0.3(x - 6.0) = 0.5x$
34. $1.9t = 0.5(4.0 - t) - 0.8$
35. $0.15 - 0.24(C - 0.50) = 0.63$
36. $27.5(5.17 - 1.44x) = 73.4$
37. $\frac{x}{2.0} = \frac{17}{6.0}$
38. $\frac{3.0}{7.0} = \frac{R}{42}$
39. $\frac{165}{223} = \frac{13V}{15}$
40. $\frac{276x}{17.0} = \frac{1360}{46.4}$

In Exercises 41–48, solve the given problems.

41. In finding the maximum operating temperature T (in °C) for a computer integrated circuit, the equation $1.1 = (T - 76)/40$ is used. Find the temperature.
42. To find the voltage V in a circuit in a TV remote-control unit, the equation $1.12V - 0.67(10.5 - V) = 0$ is used. Find V.
43. In blending two gasolines of different octanes, in order to find the number n of liters of one octane needed, the equation $0.14n + 0.06(2000 - n) = 0.09(2000)$ is used. Find n, given that 0.06 and 0.09 are exact and the first zero of 2000 is significant.
44. In order to find the distance x such that the weights are balanced on the lever shown in Fig. 1.13, the equation $210(3x) = 55.3x + 38.5(8.25 - 3x)$ must be solved. Find x. (3 is exact.)

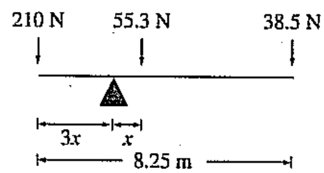

Fig. 1.13

45. A capsule contains two medications in the ratio of 5 to 2. If there are 150 mg of the first medication in the capsule, how many milligrams of the second are there?
46. A person 1.8 m tall is photographed with a 35-mm camera, and the film image is 20 mm. Under the same conditions, how tall is a person whose film image is 16 mm?
47. (W) In solving the equation $2(x - 3) + 1 = 2x - 5$, what conclusion can be made?
48. (W) In solving the equation $7 - (2 - x) = x + 2$, what conclusion can be made?

1.11 FORMULAS AND LITERAL EQUATIONS

An important application of equations is in the use of *formulas* that are found in geometry and nearly all fields of science and technology. *A* **formula** *is an equation that expresses the relationship between two or more related quantities.* For example, Einstein's famous formula $E = mc^2$ shows the equivalence of energy E to the mass m of an object and the speed of light c.

Einstein published his first paper on relativity in 1905.

NOTE ▶ If we wish to determine the value of any literal number in an expression for which we know values of the other literal numbers, we should *first solve for the required symbol and then substitute the given values.*

EXAMPLE 5 The electric resistance R (in Ω) of a resistor changes with the temperature T (in °C) according to $R = R_0 + R_0\alpha T$, where R_0 is the resistance at 0°C. For a given resistor, $R_0 = 712\ \Omega$ and $\alpha = 0.00455$/°C. Find the value of T for $R = 825\ \Omega$.

We first solve for T and then substitute the given values.

$$R = R_0 + R_0\alpha T$$
$$R - R_0 = R_0\alpha T$$
$$T = \frac{R - R_0}{\alpha R_0}$$

Now substituting, we have

$$T = \frac{825 - 712}{(0.00455)(712)}$$
$$= 34.9°\text{C} \quad \text{rounded off}$$

estimation:
$$\frac{800 - 700}{0.005(700)} = \frac{1}{0.035} = 30$$

EXERCISES 1.11

In Exercises 1–4, solve for the given letter from the indicated example of this section.

1. For the formula in Example 2, solve for a.
2. For the formula in Example 3, solve for w.
3. For the formula in Example 4, solve for T_0.
4. For the formula in Example 5, solve for α. (Do not evaluate.)

In Exercises 5–38, each of the given formulas arises in the technical or scientific area of study shown. Solve for the indicated letter.

5. $E = IR$, for R (electricity)
6. $PV = nRT$, for T (chemistry)
7. $\theta = kA + \lambda$, for λ (robotics)
8. $W = S_dT - Q$, for Q (air conditioning)
9. $Q = SLd^2$, for L (machine design)
10. $P = 2\pi Tf$, for T (mechanics)
11. $p = p_a + dgh$, for h (hydrodynamics)
12. $2Q = 2I + A + S$, for I (nuclear physics)
13. $A = \dfrac{Rt}{PV}$, for t (jet engine design)
14. $u = -\dfrac{eL}{2m}$, for L (spectroscopy)
15. $ct^2 = 0.3t - ac$, for a (medical technology)
16. $FL = P_1L - P_1d + P_2L$, for d (construction)
17. $T = \dfrac{c + d}{v}$, for d (traffic flow)
18. $L = \dfrac{N\Phi}{i}$, for Φ (electricity)
19. $\dfrac{K_1}{K_2} = \dfrac{m_1 + m_2}{m_1}$, for m_2 (kinetic energy)
20. $f = \dfrac{F}{d - F}$, for d (photography)
21. $a = \dfrac{2mg}{M + 2m}$, for M (pulleys)
22. $v = \dfrac{V(m + M)}{m}$, for M (ballistics)
23. $C_0^2 = C_1^2(1 + 2V)$, for V (electronics)
24. $A_1 = A(M + 1)$, for M (photography)
25. $P = n(p - c)$, for p (economics)
26. $T = 3(T_2 - T_1)$, for T_1 (oil drilling)
27. $F = m\left(g - \dfrac{v^2}{R}\right)$, for v^2 (circular motion)
28. $p_2 = p_1 + rp_1(1 - p_1)$, for r (population growth)
29. $Q_1 = P(Q_2 - Q_1)$, for Q_2 (refrigeration)
30. $p - p_a = dg(y_2 - y_1)$, for y_2 (pressure gauges)
31. $N = N_1T - N_2(1 - T)$, for N_1 (machine design)
32. $t_a = t_c + (1 - h)t_m$, for h (computer access time)
33. $L = \pi(r_1 + r_2) + 2x_1 + x_2$, for r_1 (pulleys)

34. $I = \dfrac{VR_2 + VR_1(1 + \mu)}{R_1R_2}$, for μ (electronics)

35. $P = \dfrac{V_1(V_2 - V_1)}{gJ}$, for V_2 (jet engine power)

36. $W = T(S_1 - S_2) - Q$, for S_2 (refrigeration)

37. $C = \dfrac{2eAk_1k_2}{d(k_1 + k_2)}$, for e (electronics)

38. $R = \dfrac{\pi r^4(p_2 - p_1)}{8nL}$, for p_1 (fluid dynamics)

In Exercises 39–44, find the indicated values.

39. The pressure p (in kPa) at a depth h (in m) below the surface of water is given by the formula $p = p_0 + kh$, where p_0 is the atmospheric pressure. Find h for $p = 205$ kPa, $p_0 = 101$ kPa, and $k = 9.80$ kPa/m.

40. A formula used in determining the total transmitted power P_t in an AM radio signal is $P_t = P_c(1 + 0.500m^2)$. Find P_c if $P_t = 680$ W and $m = 0.925$.

41. A formula relating the Fahrenheit temperature F and the Celsius temperature C is $F = \frac{9}{5}C + 32$. Find the Celsius temperature that corresponds to 90.2°F.

42. In forestry, a formula used to determine the volume V of a log is $V = \frac{1}{2}L(B + b)$, where L is the length of the log and B and b are the areas of the ends. Find b (in m^2) if $V = 1.09$ m^3, $L = 4.91$ m, and $B = 0.244$ m^2. See Fig. 1.14.

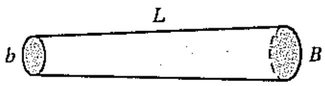

Fig 1.14

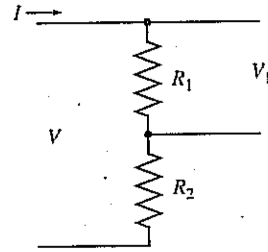

Fig. 1.15

43. The voltage V_1 across resistance R_1 (in Ω) is given by $V_1 = \dfrac{VR_1}{R_1 + R_2}$, where V is the voltage across resistances R_1 and R_2. See Fig. 1.15. Find R_2 (in Ω) if $R_1 = 3.56$ Ω, $V_1 = 6.30$ V, and $V = 12.0$ V.

44. The efficiency E of a computer multiprocessor compilation is given by $E = \dfrac{1}{q + p(1 - q)}$, where p is the number of processors and q is the fraction of the compilation that can be performed by the available parallel processors. Find p for $E = 0.66$ and $q = 0.83$.

1.12 Applied Word Problems

Many applied problems are at first word problems, and we must put them into mathematical terms for solution. Usually the most difficult part in solving a word problem is identifying the information needed for setting up the equation that leads to the solution. To do this, you must read the problem carefully to be sure that you understand all of the terms and expressions used. Following is an approach you should use.

See Appendix A, page A-3, for a variation to the method outlined in these steps. You might find it helpful. Also, some suggested study aids are briefly discussed in Appendix A.

Procedure for Solving Word Problems

1. *Read the statement of the problem.* First, read it quickly for a general overview. Then *reread* slowly and carefully, *listing the information* given.
2. *Clearly identify the unknown quantities* and then *assign an appropriate letter to represent one of them,* stating this choice clearly.
3. *Specify the other unknown quantities* in terms of the one in step 2.
4. *If possible, make a sketch* using the known and unknown quantities.
5. *Analyze the statement* of the problem and *write the necessary equation.* This is often the most difficult step because ***some of the information may be implied and not explicitly stated.*** Again, a very careful reading of the statement is necessary.
6. *Solve the equation,* clearly stating the solution.
7. *Check the solution with the original statement* of the problem.

CAUTION

EXERCISES 21.2

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 1, change $(-4, 1)$ to $(4, -1)$ and then find the equation of the line.
2. In Example 2, change $(2, -1)$ to $(-2, 1)$ and then find the equation of the line.
3. In Example 4, change the $+$ before $4x$ to $-$ and then find the slope.
4. In Example 6, change the $+$ before $2y$ to $-$ and then find the general form of the equation.

In Exercises 5–20, find the equation of each of the lines with the given properties. Sketch the graph of each line.

5. Passes through $(-3, 8)$ with a slope of 4.
6. Passes through $(-2, -1)$ with a slope of -2.
7. Passes through $(2, -5)$ and $(4, 2)$.
8. Has an x-intercept $(4, 0)$ and a y-intercept of $(0, -6)$.
9. Passes through $(1, 3)$ and has an inclination of $45°$.
10. Has a y-intercept $(0, -2)$ and an inclination of $120°$.
11. Passes through $(5.3, -2.7)$ and is parallel to the x-axis.
12. Passes through $(-4, -2)$ and is perpendicular to the x-axis.
13. Is parallel to the y-axis and is 3 units to the left of it.
14. Is parallel to the x-axis and is 4.1 units below it.
15. Is perpendicular to a line with a slope of 3 and passes through $(1, -2)$.
16. Is parallel to a line through $(-1, 2)$ and $(3, 1)$ and passes through $(1, 2)$.
17. Is parallel to a line through $(7, -1)$ and $(4, 3)$ and has a y-intercept of $(0, -2)$.
18. Is perpendicular to the line $6.0x - 2.4y - 3.9 = 0$ and passes through $(7.5, -4.7)$.
19. Has a slope of -3 and passes through the intersection of the lines $5x - y = 6$ and $x + y = 12$.
20. Passes through the point of intersection of $2x + y - 3 = 0$ and $x - y - 3 = 0$ and through the point $(4, -3)$.

In Exercises 21–28, reduce the equations to slope-intercept form and find the slope and the y-intercept. Sketch each line.

21. $4x - y = 8$	22. $2x - 3y - 6 = 0$
23. $3x + 5y - 10 = 0$	24. $4y = 6x - 9$
25. $3x - 2y - 1 = 0$	26. $4x + 2y - 5 = 0$
27. $11.2x + 1.6 = 3.2y$	28. $11.5x + 4.60y = 5.98$

In Exercises 29–36, determine whether the given lines are parallel, perpendicular, or neither.

29. $3x - 2y + 5 = 0$ and $4y = 6x - 1$
30. $8x - 4y + 1 = 0$ and $4x + 2y - 3 = 0$
31. $6x - 3y - 2 = 0$ and $x + 2y - 4 = 0$
32. $3y - 2x = 4$ and $6x - 9y = 5$
33. $5x + 2y - 3 = 0$ and $10y = 7 - 4x$
34. $48y - 36x = 71$ and $52x = 17 - 39y$
35. $4.5x - 1.8y = 1.7$ and $2.4x + 6.0y = 0.3$
36. $3.5y = 4.3 - 1.5x$ and $3.6x + 8.4y = 1.7$

In Exercises 37–56, solve the given problems. Exercises 45–5[illegible] some applications of straight lines.

37. Find k if the lines $4x - ky = 6$ and $6x + 3y + 2 = 0$ are par[illegible]
38. Find k if the lines given in Exercise 37 are perpendicula[illegible]
39. (W) Find k if the lines $3x - y = 9$ and $kx + 3y = 5$ are perpen[illegible]lar. Explain how this value is found.
40. (W) Find k such that the line through $(k, 2)$ and $(3, 1 - k)$ is perpe[illegible]cular to the line $x - 2y = 5$. Explain your method.
41. Find the slope of the line joining points on the graph o[illegible] that have x-coordinates of $-a$ and b $(a > 0, b > 0)$.
42. Show that the *intercept form* $\frac{x}{a} + \frac{y}{b} = 1$ is the equation o[illegible] with x-intercept $(a, 0)$ and y-intercept $(0, b)$.
43. Find the distance from $(4, 1)$ to the line $4x - 3y + 12 = 0$[illegible]
44. Find the acute angle between the lines $x + y = 3$ and $2x - 5y = 4$.
45. The velocity v of a box sliding down a long ramp is gi[illegible] $v = v_0 + at$, where v_0 is the initial velocity, a is the acce[illegible] and t is the time. If $v_0 = 3.35$ m/s and $v = 9.87$ m/s [illegible] $t = 4.50$ s, find v as a function of t. Sketch the graph.
46. The voltage V across part of an electric circuit is gi[illegible] $V = E - iR$, where E is a battery voltage, i is the curren[illegible] is the resistance. If $E = 6.00$ V and $V = 4.35$ V for $i = 9$[illegible] find V as a function of i. Sketch the graph (i and V may be ne[illegible]
47. The velocity of sound v increases 0.607 m/s for each incr[illegible] temperature T of 1.00°C. If $v = 343$ m/s for $T = 20.0$°C, [illegible] v as a function of T.
48. An acid solution is made from x L of a 20% solution and [illegible] 30% solution. If the final solution contains 20 L of acid, [illegible] equation relating x and y.
49. The power output P (in W) of a computer chip operating a[illegible] is proportional to $120 - T_S$, where T_S is the temperature of [illegible] roundings. If $P = 1.0$ W for $T_S = 80°C$, find the equation [illegible] P and T_S.
50. An oil-storage tank is emptied at a constant rate. A[illegible] 12 000 L remain, and at 2 P.M., 4000 L remain. If pumping [illegible] at 8 A.M., find the equation relating the number of liters [illegible] t (in h) from 8 A.M. When will the tank be empty?

A wall is 15 cm thick. At the outside, the temperature is 3°C, and at the inside, it is 23°C. If the temperature changes at a constant rate through the wall, write an equation of the temperature T in the wall as a function of the distance x from the outside to the inside of the wall. What is the meaning of the slope of the line?

The length of a rectangular solar cell is 10 cm more than the width w. Express the perimeter p of the cell as a function of w. What is the meaning of the slope of the line?

A light beam is reflected off the edge of an optic fiber at an angle of 0.0032°. The diameter of the fiber is 48 μm. Find the equation of the reflected beam with the x-axis (at the center of the fiber) and the y-axis as shown in Fig. 21.27.

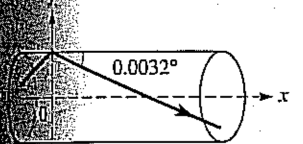

Fig. 21.27

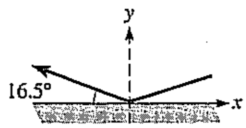

Fig. 21.28

A police report stated that a bullet caromed upward off a floor at an angle of 16.5° with the floor, as shown in Fig. 21.28. What is the equation of the bullet's path after impact?

A survey of the traffic on a particular highway showed that the number of cars passing a particular point each minute varied linearly from 6:30 A.M. to 8:30 A.M. on workday mornings. The study showed that an average of 45 cars passed the point in 1 min at 7 A.M. and that 115 cars passed in 1 min at 8 A.M. If n is the number of cars passing the point in 1 min, and t is the number of minutes after 6:30 A.M., find the equation relating n and t, and graph the equation. From the graph, determine n at 6:30 A.M. and at 8:30 A.M. What is the meaning of the slope of the line?

In a research project on cancer, a tumor was determined to weigh 30 mg when first discovered. While being treated, it grew smaller by 2 mg each month. Find the equation relating the weight w of the tumor as a function of the time t in months. Graph the equation.

In Exercises 57–60, treat the given nonlinear functions as linear functions in order to sketch their graphs. At times, this can be useful in showing certain values of a function. For example, $y = 2 + 3x^2$ can be shown as a straight line by graphing y as a function of x^2. A table of values for this graph is shown along with the corresponding graph in Fig. 21.29.

[illegible]	0	1	2	3	4	5
[illegible]	0	1	4	9	16	25
[illegible]	2	5	14	29	50	77

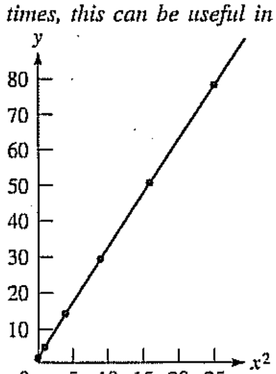

Fig. 21.29

57. The number n of memory cells of a certain computer that can be tested in t seconds is given by $n = 1200\sqrt{t}$. Sketch n as a function of $\sqrt{t}$.

58. The force F (in N) applied to a lever to balance a certain weight on the opposite side of the fulcrum is given by $F = 40/d$, where d is the distance (in m) of the force from the fulcrum. Sketch F as a function of $1/d$.

59. A spacecraft is launched such that its altitude h (in km) is given by $h = 300 + 2t^{3/2}$ for $0 \le t < 100$ s. Sketch this as a linear function.

60. The current i (in A) in a certain electric circuit is given by $i = 6(1 - e^{-t})$. Sketch this as a linear function.

In Exercises 61–64, show that the given nonlinear functions are linear when plotted on semilogarithmic or logarithmic paper. In Section 13.7, we noted that graphs on this paper often become straight lines.

61. A function of the form $y = ax^n$ is straight when plotted on logarithmic paper, since $\log y = \log a + n \log x$ is in the form of a straight line. The variables are $\log y$ and $\log x$; the slope can be found from $(\log y - \log a)/\log x = n$, and the intercept is a. (To get the slope from the graph, it is necessary to measure vertical and horizontal distances between two points. The log y-intercept is found where $\log x = 0$, and this occurs when $x = 1$.) Plot $y = 3x^4$ on logarithmic paper to verify this analysis.

62. A function of the form $y = a(b^x)$ is a straight line on semilogarithmic paper, since $\log y = \log a + x \log b$ is in the form of a straight line. The variables are $\log y$ and x, the slope is $\log b$, and the intercept is a. (To get the slope from the graph, we calculate $(\log y - \log a)/x$ for some set of values x and y. The intercept is read directly off the graph where $x = 0$.) Plot $y = 3(2^x)$ on semilogarithmic paper to verify this analysis.

63. If experimental data are plotted on logarithmic paper and the points lie on a straight line, it is possible to determine the function (see Exercise 61). The following data come from an experiment to determine the functional relationship between the pressure p and the volume V of a gas undergoing an adiabatic (no heat loss) change. From the graph on logarithmic paper, determine p as a function of V.

V (m^3)	0.100	0.500	2.00	5.00	10.0
p (kPa)	20.1	2.11	0.303	0.0840	0.0318

64. If experimental data are plotted on semilogarithmic paper, and the points lie on a straight line, it is possible to determine the function (see Exercise 62). The following data come from an experiment designed to determine the relationship between the voltage across an inductor and the time, after the switch is opened. Determine v as a function of t.

v (V)	40	15	5.6	2.2	0.8
t (ms)	0.0	20	40	60	80

In Section 14.1, we noted that the equation of a circle does not represent a *function* since there are two values of y for most values of x in the domain. In fact, *it might be necessary to use the quadratic formula to find the two functions to enter into a graphing calculator* in order to view the curve. This is illustrated in the following example.

NOTE ▸

EXAMPLE 7 Display the graph of the circle $3x^2 + 3y^2 + 6y - 20 = 0$ on a graphing calculator.

To fit the form of a quadratic equation in y, we write

$$3y^2 + 6y + (3x^2 - 20) = 0$$

Now, using the quadratic formula to solve for y, we let

$$a = 3 \quad b = 6 \quad c = 3x^2 - 20$$

Therefore,

$$y = \frac{-6 \pm \sqrt{6^2 - 4(3)(3x^2 - 20)}}{2(3)}$$

which means we get the *two functions*

$$y_1 = \frac{-6 + \sqrt{276 - 36x^2}}{6} \text{ and } y_2 = \frac{-6 - \sqrt{276 - 36x^2}}{6}$$

which are entered into the calculator to get the view shown in Fig. 21.38.

The *window* values were chosen so that the length along the x-axis is about 1.5 times that along the y-axis so as to have less distortion in the circle. There may be gaps at the left and right sides of the circle.

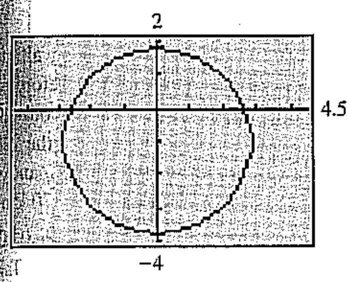

Fig. 21.38

EXERCISES 21.3

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 1, change $(y + 2)^2$ to $(y + 1)^2$ and then find the center and radius. Sketch the circle.
2. In Example 2, change $(2, 1)$ to $(-2, 1)$ and then find the equation of the circle. Sketch the circle.
3. In Example 5, change the + before $8y$ to − and then find the center and radius. Sketch the circle.
4. In Example 7, change the + before $6y$ to − and then display the graph on a graphing calculator.

In Exercises 5–8, determine the center and the radius of each circle.

5. $(x - 2)^2 + (y - 1)^2 = 25$
6. $(x - 3)^2 + (y + 4)^2 = 49$
7. $4(x + 1)^2 + 4y^2 = 9$
8. $9x^2 + 9(y - 6)^2 = 64$

In Exercises 9–24, find the equation of each of the circles from the given information.

9. Center at $(0, 0)$, radius 3
10. Center at $(0, 0)$, radius 1
11. Center at $(2, 2)$, radius 4
12. Center at $\left(\frac{3}{2}, -2\right)$, radius $\frac{5}{2}$
13. Center at $(12, -15)$, radius 18
14. Center at $(-3, -5)$, radius $2\sqrt{3}$
15. The origin and $(-6, 8)$ are ends of a diameter
16. The points $(3, 8)$ and $(-3, 0)$ are the ends of a diameter.
17. Concentric with the circle $(x - 2)^2 + (y - 1)^2 = 4$ and passes through $(4, -1)$
18. Concentric with the circle $(x + 1)^2 + (y - 4)^2 = 9$ and passes through $(-2, 3)$
19. Center at $(-3, 5)$, tangent to the x-axis
20. Center at $(2, -4)$, tangent to the y-axis
21. Tangent to both axes and the lines $y = 4$ and $x = 4$
22. Tangent to both axes, radius 4, in the second quadrant
23. Center at the origin, tangent to the line $x + y = 2$
24. Center at $(5, 12)$, tangent to the line $y = 2x - 3$

In Exercises 25–36, determine the center and radius of each circle. Sketch each circle.

25. $x^2 + (y - 3)^2 = 4$
26. $(x - 2)^2 + (y + 3)^2 = 49$
27. $4(x + 1)^2 + 4(y - 5)^2 = 81$

28. $2(x + 4)^2 + 2(y + 3)^2 = 25$
29. $x^2 + y^2 - 2x - 8 = 0$
30. $x^2 + y^2 - 4x - 6y - 12 = 0$
31. $x^2 + y^2 + 4.20x - 2.60y = 3.51$
32. $x^2 + y^2 + 22x + 14y = 26$
33. $4x^2 + 4y^2 - 16y = 9$
34. $9x^2 + 9y^2 + 18y = 7$
35. $2x^2 + 2y^2 - 4x - 8y - 1 = 0$
36. $3x^2 + 3y^2 - 12x + 4 = 0$

In Exercises 37–40, determine whether the circles with the given equations are symmetric to either axis or to the origin.

37. $x^2 + y^2 = 100$
38. $x^2 + y^2 - 4x - 5 = 0$
39. $3x^2 + 3y^2 + 24y = 8$
40. $5x^2 + 5y^2 - 10x + 20y = 3$

In Exercises 41–56, solve the given problems.

41. Determine whether the circle $x^2 - 6x + y^2 - 7 = 0$ crosses the x-axis.
42. Find the points of intersection of the circle $x^2 + y^2 - x - 3y = 0$ and the line $y = x - 1$.
43. (W) Find the locus of a point $P(x, y)$ that moves so that its distance from $(2, 4)$ is twice its distance from $(0, 0)$. Describe the locus.
44. (W) Find the equation of the locus of a point $P(x, y)$ that moves so that the line joining it and $(2, 0)$ is always perpendicular to the line joining it and $(-2, 0)$. Describe the locus.
45. Use a graphing calculator to view the circle $x^2 + y^2 + 5y - 4 = 0$.
46. Use a graphing calculator to view the circle $2x^2 + 2y^2 + 2y - x - 1 = 0$.
47. (W) What type of graph is represented by the equations
(a) $y = \sqrt{9 - (x - 2)^2}$? (b) $y = -\sqrt{9 - (x - 2)^2}$?
(c) Are the equations in parts (a) and (b) functions? Explain.
48. What type of graph is represented by the equations
(a) $x^2 + (y - 1)^2 = 0$? (b) $x^2 + (y - 1)^2 = -1$?
49. In a hoisting device, two of the pulley wheels may be represented by $x^2 + y^2 = 14.5$ and $x^2 + y^2 - 19.6y + 86.0 = 0$. How far apart (in cm) are the wheels?
50. The design of a machine part shows it as a circle represe the equation $x^2 + y^2 = 42.5$, with a circular hole represe $x^2 + y^2 + 3.06y - 1.24 = 0$ cut out. What is the least (in cm) from the edge of the hole to the edge of the machi
51. A wire is rotating in a circular path through a magnetic induce an electric current in the wire. The wire is rot 60.0 Hz with a constant velocity of 37.7 m/s. Taking the ori the center of the circle of rotation, find the equation of the the wire.
52. A communications satellite remains stationary at an alti 36 200 km over a point on the earth's equator. It therefore once each day about the earth's center. Its velocity is const the horizontal and vertical components, v_H and v_V, of the constantly change. Show that the equation relating v_H and km/h) is that of a circle. The radius of the earth is 6370 km
53. Find the equation describing the rim of a circular porthole diameter if the top is 2.0 m below the surface of the water. origin at the water surface directly above the center of the
54. An earthquake occurred 37° north of east of a seismic station. If the tremors travel at 4.8 km/s and were recorded later at the station, find the equation of the circle that the tremor recorded at the station. Take the station to be at the ter of the coordinate system.
55. In analyzing the strain on a beam, *Mohr's circle* is often form it, normal strain is plotted as the x-coordinate and strain is plotted as the y-coordinate. The center of the circ midway between the minimum and maximum values of strain on the x-axis. Find the equation of Mohr's circle if the mum normal strain is 100×10^{-6} and the maximum normal is 900×10^{-6} (strain is unitless). Sketch the graph.
56. An architect designs a Norman window, which has the form semicircle surmounted on a rectangle, as in Fig. 21.39. Find area (in m²) of the window if the circular part is on the $x^2 + y^2 - 3.00y + 1.25 = 0$.

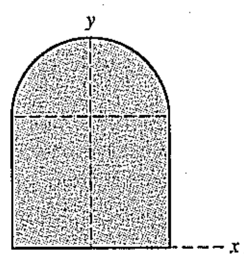

Fig. 21.39

21.4 THE PARABOLA

In Chapter 7, we showed that the graph of a quadratic function is a *parabola*. We define the parabola more generally and find the general form of its equation.

A **parabola** *is defined as the locus of a point $P(x, y)$ that moves so that it is equidistant from a given line (the* **directrix***) and a given point (the* **focus***). The line through the focus that is perpendicular to the directrix is the* **axis** *of the parabola. The point midway between the focus and directrix is the* **vertex.**