

If one of the trigonometric functions is known, it is possible to find the values of the other functions. The following example illustrates the method.

**EXAMPLE 5** If we know that  $\sin \theta = 3/7$  and that  $\theta$  is a first-quadrant angle, we know the ratio of the ordinate to the radius vector ( $y$  to  $r$ ) is 3 to 7. Therefore, the point on the terminal side for which  $y = 3$  can be found by use of the Pythagorean theorem. The  $x$ -value for this point is

$$x = \sqrt{7^2 - 3^2} = \sqrt{49 - 9} = \sqrt{40} = 2\sqrt{10}$$

Therefore, the point  $(2\sqrt{10}, 3)$  is on the terminal side, as shown in Fig. 4.17.

Therefore, using the values  $x = 2\sqrt{10}$ ,  $y = 3$ , and  $r = 7$ , we have the other trigonometric functions of  $\theta$ . They are

$$\cos \theta = \frac{2\sqrt{10}}{7} \quad \tan \theta = \frac{3}{2\sqrt{10}} \quad \cot \theta = \frac{2\sqrt{10}}{3} \quad \sec \theta = \frac{7}{2\sqrt{10}} \quad \csc \theta = \frac{7}{3}$$

These values are *exact*. *Approximate* decimal values found on a calculator are

$$\begin{array}{lll} \cos \theta = 0.9035 & \tan \theta = 0.4743 & \cot \theta = 2.108 \\ \sec \theta = 1.107 & \csc \theta = 2.333 & \end{array}$$

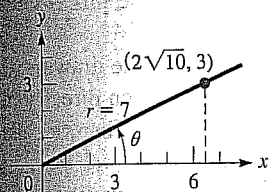


Fig. 4.17

## EXERCISES 4.2

In Exercises 1 and 2, answer the given questions about the indicated examples of this section.

- In Example 3, if the point  $(4, 3)$  replaces the point  $(3, 4)$ , what are the values?
- In Example 5, if  $4/7$  replaces  $3/7$ , what are the values?

In Exercises 3–16, find values of the trigonometric functions of the angle (in standard position) whose terminal side passes through the given points. For Exercises 3–14, give answers in exact form. For Exercises 15 and 16, the coordinates are approximate.

- |                      |                      |                     |                        |
|----------------------|----------------------|---------------------|------------------------|
| 3. $(6, 8)$          | 4. $(5, 12)$         | 5. $(15, 8)$        | 6. $(24, 7)$           |
| 7. $(0.9, 4.0)$      | 8. $(3.2, 6.0)$      | 9. $(1, \sqrt{15})$ | 10. $(\sqrt{3}, 2)$    |
| 11. $(1, -1)$        | 12. $(6, 5)$         | 13. $(5, 2)$        | 14. $(1, \frac{1}{2})$ |
| 15. $(0.687, 0.943)$ | 16. $(37.65, 21.87)$ |                     |                        |

In Exercises 17–24, find the values of the indicated functions. In Exercises 17–20, give answers in exact form. In Exercises 21–24, the values are approximate.

- Given  $\cos \theta = 12/13$ , find  $\sin \theta$  and  $\cot \theta$ .
- Given  $\sin \theta = 1/2$ , find  $\cos \theta$  and  $\csc \theta$ .
- Given  $\tan \theta = 2$ , find  $\sin \theta$  and  $\sec \theta$ .
- Given  $\sec \theta = \sqrt{5}/2$ , find  $\tan \theta$  and  $\cos \theta$ .
- Given  $\sin \theta = 0.750$ , find  $\cot \theta$  and  $\csc \theta$ .
- Given  $\cos \theta = 0.326$ , find  $\sin \theta$  and  $\tan \theta$ .
- Given  $\cot \theta = 0.254$ , find  $\cos \theta$  and  $\tan \theta$ .
- Given  $\csc \theta = 1.20$ , find  $\sec \theta$  and  $\cos \theta$ .

In Exercises 25–28, each point listed is on the terminal side of an angle. Show that each of the indicated functions is the same for each of the points.

- $(3, 4)$ ,  $(6, 8)$ ,  $(4.5, 6)$ ,  $\sin \theta$  and  $\tan \theta$
- $(5, 12)$ ,  $(15, 36)$ ,  $(7.5, 18)$ ,  $\cos \theta$  and  $\cot \theta$
- $(0.3, 0.1)$ ,  $(9, 3)$ ,  $(33, 11)$ ,  $\tan \theta$  and  $\sec \theta$
- $(0.4, 0.3)$ ,  $(8, 6)$ ,  $(36, 27)$ ,  $\csc \theta$  and  $\cos \theta$

In Exercises 29–36, answer the given questions.

- If  $\tan \theta = 3/4$ , what is the value of  $\sin^2 \theta + \cos^2 \theta$ ?  
[ $\sin^2 \theta = (\sin \theta)^2$ ]
- If  $\sin \theta = 2/3$ , what is the value of  $\sec^2 \theta - \tan^2 \theta$ ?
- If  $y = \sin \theta$ , what is  $\cos \theta$  in terms of  $y$ ?
- If  $x = \cos \theta$ , what is  $\tan \theta$  in terms of  $x$ ?
- What is  $x$  if  $(x + 1, 4)$  and  $(-2, 6)$  are on the same terminal side of a standard position angle?
- What is  $x$  if  $(2, 5)$  and  $(7, x)$  are on the same terminal side of a standard position angle?
- From the definitions of the trigonometric functions, it can be seen that  $\csc \theta$  is the reciprocal of  $\sin \theta$ . What function is the reciprocal of  $\cos \theta$ ?
- Refer to the definitions of the trigonometric functions in Eqs. (4.1). Is the quotient of one of the functions divided by  $\cos \theta$  equal to  $\tan \theta$ ? Explain.

The following example illustrates the use of the value of a trigonometric function in an applied problem. We consider various types of applications later in the chapter.

**EXAMPLE 8** When a rocket is launched, its horizontal velocity  $v_x$  is related to the velocity  $v$  with which it is fired by the equation  $v_x = v \cos \theta$  (which means  $v(\cos \theta)$ , but does *not* mean  $\cos \theta v$ , which is the same as  $\cos(\theta v)$ ). Here,  $\theta$  is the angle between the horizontal and the direction in which it is fired (see Fig. 4.24). Find  $v_x$  if  $v = 1250$  m/s and  $\theta = 36.0^\circ$ .

Substituting the given values of  $v$  and  $\theta$  in  $v_x = v \cos \theta$ , we have

$$\begin{aligned} v_x &= 1250 \cos 36.0^\circ \\ &= 1010 \text{ m/s} \end{aligned}$$

Therefore, the horizontal velocity is 1010 m/s.

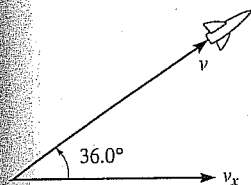


Fig. 4.24

### EXERCISES 4.3

In Exercises 1–4, make the given changes in the indicated examples of this section and then find the indicated values.

- In Example 4, change  $\cos \theta$  to  $\sin \theta$  and then find the angle.
- In Example 5, change  $\sec 27.82^\circ$  to  $\csc 27.82^\circ$  and then find the value.
- In Example 6, change 0.354 to 0.345 and then find the angle.
- In Example 7, change  $\sin \theta$  to  $\tan \theta$  and then find the value.

In Exercises 5–8, use a protractor to draw the given angle. Measure off 10 units (centimeters are convenient) along the radius vector. Then measure the corresponding values of  $x$  and  $y$ . From these values, determine the trigonometric functions of the angle.

- $40^\circ$
- $75^\circ$
- $15^\circ$
- $53^\circ$

In Exercises 9–24, find the values of the trigonometric functions. Round off results according to the table following Example 4.

- |                        |                        |
|------------------------|------------------------|
| 9. $\sin 22.4^\circ$   | 10. $\cos 72.5^\circ$  |
| 11. $\tan 57.6^\circ$  | 12. $\sin 36.0^\circ$  |
| 13. $\cos 15.71^\circ$ | 14. $\tan 8.653^\circ$ |
| 15. $\sin 84^\circ$    | 16. $\cos 47^\circ$    |
| 17. $\cot 67.78^\circ$ | 18. $\csc 22.81^\circ$ |
| 19. $\sec 50.4^\circ$  | 20. $\cot 41.8^\circ$  |
| 21. $\csc 49.3^\circ$  | 22. $\sec 7.8^\circ$   |
| 23. $\cot 85.96^\circ$ | 24. $\csc 76.30^\circ$ |

In Exercises 25–40, find  $\theta$  for each of the given trigonometric functions. Round off results according to the table following Example 4.

- |                            |                            |
|----------------------------|----------------------------|
| 25. $\cos \theta = 0.3261$ | 26. $\tan \theta = 2.470$  |
| 27. $\sin \theta = 0.9114$ | 28. $\cos \theta = 0.0427$ |

- |                             |                            |
|-----------------------------|----------------------------|
| 29. $\tan \theta = 0.207$   | 30. $\sin \theta = 0.109$  |
| 31. $\cos \theta = 0.65007$ | 32. $\tan \theta = 5.7706$ |
| 33. $\csc \theta = 1.245$   | 34. $\sec \theta = 2.045$  |
| 35. $\cot \theta = 0.1443$  | 36. $\csc \theta = 1.012$  |
| 37. $\sec \theta = 3.65$    | 38. $\cot \theta = 2.08$   |
| 39. $\csc \theta = 3.262$   | 40. $\cot \theta = 0.1519$ |

In Exercises 41–44, use a calculator to verify the given relationships or statements. [ $\sin^2 \theta = (\sin \theta)^2$ ]

- |  |   |
|--|---|
| 41. $\frac{\sin 43.7^\circ}{\cos 43.7^\circ} = \tan 43.7^\circ$                                | 42. $\sin^2 77.5^\circ + \cos^2 77.5^\circ = 1$ |
| 43. $\tan 70^\circ = \frac{\tan 30^\circ + \tan 40^\circ}{1 - (\tan 30^\circ)(\tan 40^\circ)}$ |   |
| 44. $\sin 78.4^\circ = 2(\sin 39.2^\circ)(\cos 39.2^\circ)$                                    |   |

**(W)** In Exercises 45–48, explain why the given statements are true for an acute angle  $\theta$ .

- $\sin \theta$  is always between 0 and 1.
- $\tan \theta$  can equal any positive real number.
- $\cos \theta$  decreases in value from  $0^\circ$  to  $90^\circ$ .
- The value of  $\sec \theta$  is never less than 1.

In Exercises 49–52, find the values of the indicated trigonometric functions.

- Find  $\sin \theta$ , given  $\tan \theta = 1.936$ .
- Find  $\cos \theta$ , given  $\sin \theta = 0.6725$ .
- Find  $\tan \theta$ , given  $\sec \theta = 1.3698$ .
- Find  $\csc \theta$ , given  $\cos \theta = 0.1063$ .