

Bonus Quiz 2

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Evaluate the integral if it converges.

$$\int_1^{\infty} \frac{1}{(2x-1)^{3/2}} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{(2x-1)^{3/2}} dx$$

$$\begin{aligned} u &= 2x-1 & u(1) &= 2(1)-1 = 1 \\ du &= 2dx & u(b) &= 2b-1 \\ \frac{du}{2} &= dx \end{aligned}$$

$$\stackrel{①, ②}{=} \lim_{b \rightarrow \infty} \int_1^{2b-1} \frac{1}{u^{3/2}} \frac{du}{2}$$

$$= \frac{1}{2} \lim_{b \rightarrow \infty} \left[2u^{-1/2} \right]_1^{2b-1}$$

$$= \frac{1}{2} \lim_{b \rightarrow \infty} \left[\frac{-2}{\sqrt{2b-1}} + \frac{2}{\sqrt{1}} \right]$$

$$= \frac{1}{2} \frac{2}{1} = 1$$

Question 2. (5 marks) Verify that

$$y = \frac{C}{x} \quad (3)$$

$$y' = \frac{-C}{x^2} \quad (1)$$

is a general solution of

$$y' + \left(\frac{1}{x}\right)y = 0 \quad (2)$$

sub (1), (3) into (2)

and find the particular solution that satisfies $y(1) = 1$.

$$\begin{aligned} \frac{-C}{x^2} + \frac{1}{x} \left(\frac{C}{x} \right) &\stackrel{?}{=} 0 \\ &\stackrel{?}{=} 0 \\ \frac{-C}{x^2} + \frac{C}{x^2} &= 0 \end{aligned}$$

$$\begin{aligned} \text{if } y(1) &= 1 \\ \frac{C}{1} &= 1 \\ C &= 1 \end{aligned}$$

$$0 = 0 \quad \text{so it is}$$

a general solution.

$$y = \frac{1}{x}$$