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Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Integrate the following indefinite integral:

$\int e^x \cos(2x)\,dx = I$ (u = $e^x\cos(2x)$, dv = dx)

$u = \cos(2x) \Rightarrow du = -\sin(2x)\,2\,dx$

$v = e^x \Leftarrow dv = e^x dx$

$I = uv - \int v\,du$

$I = e^x\cos(2x) - \int e^x(-\sin(2x)\,2)\,dx$

$I = e^x\cos(2x) + 2\int e^x \sin(2x)\,dx$ (u = $\sin(2x)$, dv = $e^x dx$)

$u = \sin(2x)$ $du = \cos(2x)2dx$

$v = e^x$ $dv = e^x dx$

$I = e^x\cos(2x) + 2[uv - \int v\,du]$

$I = e^x\cos(2x) + 2[e^x\sin(2x) - \int e^x\cos(2x)\,2dx]$

$I = e^x\cos(2x) + 2e^x\sin(2x) - 4\int e^x\cos(2x)\,dx$ ($\int e^x\cos(2x)\,dx = I$)

$I = e^x\cos(2x) + 2e^x\sin(2x) - 4I + C$

$5I = e^x\cos(2x) + 2e^x\sin(2x) + C$

$I = \frac{1}{5}[e^x\cos(2x) + 2e^x\sin(2x)] + C$

Question 2. *(5 marks)* Integrate the following definite integral:

$\int_0^1 3ye^{2y}\,dy$

$u = y \Rightarrow du = dy$

$v = \frac{e^{2y}}{2} \Leftarrow dv = e^{2y}dy$

$= 3\int_0^1 ye^{2y}\,dy$

$= 3\left[[uv]_0^1 - \int_0^1 v\,du\right]$

$= 3\left[\left[y\frac{e^{2y}}{2}\right]_0^1 - \int_0^1 \frac{e^{2y}}{2}dy\right]$

$= 3\left[\left[1\frac{e^2}{2} - 0\frac{e^0}{2}\right] - \left[\frac{e^{2y}}{4}\right]_0^1\right]$

$= 3\left[\frac{e^2}{2} - \left[\frac{e^{2(1)}}{4} - \frac{e^0}{4}\right]\right]$

$= \frac{3e^2}{2} - \frac{3e^2}{4} + \frac{3}{4}$

$= \frac{3e^2+3}{4}$

Question 3. *(5 marks)* Integrate the following indefinite integral:

$\int \frac{x^4+2x+1}{x+2}\,dt$

$= \int x^3 - 2x^2 + 4x - 6 + \frac{13}{x+2}\,dx$

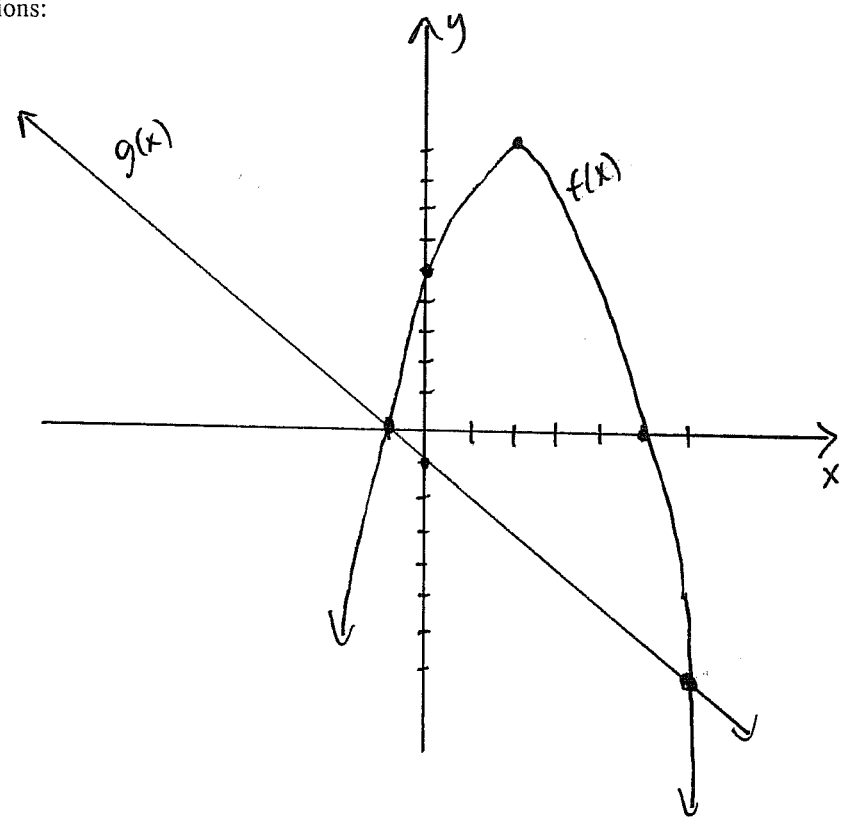
$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^2}{2} - 6x + 13\ln|x+2| + C$

$$
\begin{array}{r|l}
 & x^3 - 2x^2 + 4x - 6 \\
\hline
x+2 & x^4 + 0x^3 + 0x^2 + 2x + 1 \\
 & -(x^4 + 2x^3) \\
\hline
 & -2x^3 + 0x^2 \\
 & -(-2x^3 - 4x^2) \\
\hline
 & 4x^2 + 2x \\
 & -(4x^2 + 8x) \\
\hline
 & -6x + 1 \\
 & -(-6x - 12) \\
\hline
 & 13
\end{array}
$$

Question 4. *(10 marks)* Sketch the graph of the following functions:

$f(x) = -x^2 + 4x + 5$ and $g(x) = -x - 1$.

Then find the area bounded by the two functions.



$g(x)$ has x-int $(-1, 0)$
y-int $(0, -1)$

$f(x)$ has vertex:

$$f(x) = -[x^2 - 4x - 5]$$
$$= -[x^2 - 4x + 4 - 4 - 5]$$
$$= -[(x^2 - 4x + 4) - 9]$$
$$= -[(x-2)^2 - 9]$$
$$= -(x-2)^2 + 9$$

∴ vertex at $(2, 9)$
has y-int $(0, 5)$
has x-int:

$$0 = -x^2 + 4x + 5$$
$$0 = x^2 - 4x - 5$$
$$0 = (x-5)(x+1)$$
$$x = 5 \quad x = -1$$

Intersection of $f(x)$ and $g(x)$

$$f(x) = g(x)$$
$$-x - 1 = -x^2 + 4x + 5$$
$$0 = x^2 - 5x - 6$$
$$0 = (x-6)(x+1)$$
$$x = 6 \quad x = -1$$

∴ Area $= \int_{-1}^{6} -x^2 + 4x + 5 - [-x - 1]\, dx$

$$= \int_{-1}^{6} -x^2 + 5x + 6\, dx$$

$$= \left[\frac{-x^3}{3} + \frac{5x^2}{2} + 6x \right]_{-1}^{6}$$

$$= \frac{-6^3}{3} + \frac{5(6)^2}{2} + 6(6) - \left[\frac{-(-1)^3}{3} + \frac{5(-1)^2}{2} + 6(-1) \right]$$

$$= \frac{-216}{3} + \frac{180}{2} + 36 - \frac{1}{3} - \frac{5}{2} + 6$$

$$= \frac{343}{6}$$

Question 5. *(10 marks)* The bookstore "L'Insoumise" has studied the demand and supply equations for their in house published book The Family by Ba Jin. They have determined that the demand equation is given by

$$p = \frac{400}{x+4}$$

and ~~demand~~ supply equation is given by

$$p = \frac{x+38}{2}.$$

Find the consumers' surplus and producers' surplus.

Lets find the market equilibrium

$$\frac{400}{x+4} = \frac{x+38}{2}$$

$$800 = (x+38)(x+4)$$

$$800 = x^2 + 42x + 152$$

$$0 = x^2 + 42x - 648$$

$$0 = (x-12)(x+54)$$

$x = 12$ $\quad$ $x = \cancel{-54}$ not valid

i.e. $\bar{x} = 12$ and

$$\bar{p} = \frac{12+38}{2} = 25$$

$$PS = \int_0^{12} 25 - \frac{x+38}{2}\, dx$$

$$= \int_0^{12} -\frac{x}{2} + 6\, dx$$

$$= \left[-\frac{x^2}{4} + 6x\right]_0^{12}$$

$$= -\frac{12^2}{4} + 6(12)$$

$$= 36.00\$$$

$$CS = \int_0^{12} \frac{400}{x+4} - 25\, dx$$

$$= \left[400 \ln|x+4| - 25x\right]_0^{12}$$

$$= 400 \ln|12+4| - 25(12) - [400 \ln 4 - 25(0)]$$

$$= 400 \ln(16) - 300 - 400 \ln 4$$

$$= 400 \ln 4 - 300$$

$$\doteq 254.52\$$$

Question 6. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{x^2+x+1}{(x+1)(x-2)^2}\,dx$$

$$\frac{x^2+x+1}{(x+1)(x-2)^2} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$x^2+x+1 = \frac{A(x+1)(x-2)^2}{(x+1)} + \frac{B(x+1)(x-2)^2}{(x-2)} + \frac{C(x+1)(x-2)^2}{(x-2)^2}$$

$$x^2+x+1 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

Let $x=-1$

$$(-1)^2+(-1)+1 = A(-1-2)^2 + B(-1+1)(-1-2) + C(-1+1)$$

$$1 = 9A$$

$$\frac{1}{9} = A$$

Let $x=0$

$$0^2+0+1 = A(0-2)^2 + B(0+1)(0-2) + C(0+1)$$

$$1 = 4A - 2B + C$$

$$1 = \frac{4}{9} - 2B + C$$

$$\frac{5}{9} = -2B + C \quad (1)$$

Let $x=2$

$$2^2+2+1 = A(2-2)^2 + B(2+1)(2-2) + C(2+1)$$

$$7 = 3C$$

$$\frac{7}{3} = C \quad (2)$$

sub (2) into (1)

$$2B = -\frac{5}{9} + C$$

$$2B = \frac{-5}{9} + \frac{7}{3}$$

$$B = \frac{8}{9}$$

$$\therefore \int \frac{1/9}{x+1} + \frac{-1/12}{x-2} + \frac{7/3}{(x-2)^2}\,dx$$

$$= \frac{1}{9}\ln|x+1| - \frac{8}{9}\ln|x-2| - \frac{7}{3}(x-2)^{-1} + C$$