

Name: _____
Student ID: _____

Test 3

This test is graded out of 41 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Integrate the following indefinite integral (using the Table of Integrals):

$$\int \frac{3e^x}{2 + 3e^{\frac{1}{2}x}} dx$$

Question 2. (5 marks) Evaluate the following definite integral (using the Table of Integrals), simplify completely:

$$\int_3^5 \frac{1}{x^2\sqrt{x^2-9}} dx$$

Question 3. Evaluate the following limits:

a. (3 marks)

$$\lim_{x \rightarrow 0} \frac{\sin x}{1 - \cos x}$$

b. (5 marks)

$$\lim_{x \rightarrow 1} \frac{3e^{1-x} + 2\ln x - 3x}{x^2 - 1}$$

Question 4. (5 marks) Evaluate the following improper integral:

$$\int_1^{\infty} \frac{1}{(1+2x)^{7/2}} dx$$

Question 5. (3 marks) Evaluate the following improper integral:

$$\int_{-\infty}^{-1} \frac{101}{x} dx$$

Question 6. (10 marks) Use the trapezoidal rule with $n = 4$ to approximate the integral and compare to the exact value of the integral.

$$\int_1^2 x \ln x \, dx$$

Question 7. (5 marks) Verify that

$$y'' - 3y' + 2y = e^x$$

is the general solution of the differential equation

$$y = C_1 e^x + C_2 e^{2x} - x e^x.$$

Then find the particular solution that satisfy the initial condition $y(0) = 0$ and $y'(0) = 0$.

Bonus Question. (3 marks) Evaluate the following improper integral.

$$\int_1^{\infty} x e^{-x} dx$$

Table of Integrals

1. $\int \frac{u}{a+bu} du = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$
2. $\int \frac{u^2}{a+bu} du = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$
3. $\int \frac{u}{(a+bu)^2} du = \frac{1}{b^2} \left(\frac{a}{a+bu} + \ln |a + bu| \right) + C$
4. $\int u\sqrt{a+bu} du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{\frac{3}{2}} + C$
5. $\int \frac{u}{\sqrt{a+bu}} du = \frac{2}{3b^2} (bu - 2a)\sqrt{a + bu} + C$
6. $\int \frac{1}{u\sqrt{a+bu}} du = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C$ if $a > 0$
7. $\int \sqrt{a^2 + u^2} du = \frac{u}{a} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln |u + \sqrt{a^2 + u^2}| + C$
8. $\int u^2 \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + 2u^2} - \frac{a^4}{8} \ln |u + \sqrt{a^2 + u^2}| + C$
9. $\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln |u + \sqrt{a^2 + u^2}| + C$
10. $\int \frac{1}{a^2 + u^2} du = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
11. $\int \frac{1}{u^2 \sqrt{a^2 + u^2}} du = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
12. $\int \frac{1}{(a^2 + u^2)^{3/2}} du = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$
13. $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
14. $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$
15. $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$
16. $\int \frac{1}{\sqrt{u^2 - a^2}} du = \ln |u + \sqrt{u^2 - a^2}| + C$
17. $\int \frac{1}{u^2 \sqrt{u^2 - a^2}} du = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$
18. $\int \frac{1}{(u^2 - a^2)^{3/2}} du = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$
19. $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
20. $\int \frac{1}{u\sqrt{a^2 - u^2}} du = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
21. $\int \frac{1}{u^2 \sqrt{a^2 - u^2}} du = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$
22. $\int \frac{1}{(a^2 - u^2)^{3/2}} du = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$