

Test 4

This test is graded out of 44 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Find the general solution of the following differential equation.

$$y' = \frac{y^2 x e^{x^2}}{1 + y^2}$$

Question 2. (5 marks) Find the solution to the following following initial value problem:

$$y' = 5xy - 3x$$

where $y(0) = 1$.

Question 3. (5 marks) Find the third Taylor Polynomial of $f(x) = xe^{-3x} + \cos 2x$ at $x = 0$

Question 4. Determine the convergence or divergence of the following sequences.

a. (2 marks)

$$a_n = \frac{\ln n}{n}$$

b. (2 marks)

$$a_n = \frac{2n^2 - n - 1}{3n^2 + 3n + 101}$$

Question 5. (5 marks) Determine if the following series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

Question 6. (5 marks) Determine if the following series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{5^n + 1}{7^{n+1}}$$

Question 7. (5 marks) Determine if the following series converges or diverges, justify your answer and state the test used.

$$\sum_{n=1}^{\infty} \frac{e^{-\ln n}}{n}$$

Question 8. (5 marks) Determine if the following series converges or diverges, justify your answer and state the test used.

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^2 + 1}}$$

Question 9. (5 marks) Determine if the following series converges or diverges, justify your answer and state the test used.

$$\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^5 + 1}}$$

Bonus. (3 marks) Determine if the following series converges or diverges, justify your answer and state the test used.

$$\sum_{n=0}^{\infty} \frac{n}{e^n}$$