Dawson College: **Calculus II**: 201-NYB-05-C2: Fall 2009

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Quiz 10

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §9.3 #15
Use the Integral Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$$

Let $f(x) = \dfrac{\arctan x}{x^2+1}$

- $f(x)$ is positive for $x \geq 1$ since $\arctan x \geq \frac{\pi}{4}$ for $x \geq 1$ and x^2+1 is always positive.
- $f(x)$ is continuous.
- $f'(x) = \dfrac{\left[\frac{1}{\cancel{1+x^2}}\cancel{(x^2+1)} - (\arctan x)(2x)\right]}{(x^2+1)^2} = \dfrac{1 - 2x\arctan x}{(x^2+1)^2} < 0$ since

$(x^2+1)^2 \geq 1$ and $\arctan x \geq \frac{\pi}{4}$ for $x \geq 1$.

Lets use the integral test

$$\int_1^{\infty} \frac{\arctan x}{x^2+1}\,dx = \lim_{b\to\infty} \int_1^b \frac{\arctan x}{x^2+1}$$

$u = \arctan x$

$du = \dfrac{dx}{(x^2+1)}$

$(x^2+1)\,du = dx$

$u(1) = \frac{\pi}{4}$

$u(b) = \arctan b$

$$= \lim_{b\to\infty} \int_{\frac{\pi}{4}}^{\arctan b} \frac{u}{\cancel{x^2+1}}\cancel{(x^2+1)}\,du$$

$$= \lim_{b\to\infty} \int_{\frac{\pi}{4}}^{\arctan b} u\,du$$

$$= \lim_{b\to\infty} \left[\frac{u^2}{2}\right]_{\frac{\pi}{4}}^{\arctan b}$$

$$= \lim_{b\to\infty} \left[\frac{(\overset{\pi/2}{\cancel{\arctan b}})^2}{2} - \frac{\left(\frac{\pi}{4}\right)^2}{2}\right]$$

$$= \frac{\left(\frac{\pi}{2}\right)^2}{2} - \frac{\left(\frac{\pi}{4}\right)^2}{2}$$

$$= \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$ converges since $\displaystyle\int_1^{\infty} \frac{\arctan x}{x^2+1}\,dx$ converges

Question 2. *(5 marks)* §9.2 #48
Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} [(0.7)^n + (0.9)^n]$$

$$= \sum_{n=1}^{\infty} \left[\left(\frac{7}{10}\right)^n + \left(\frac{9}{10}\right)^n\right]$$

$$= \sum_{n=1}^{\infty} \left(\frac{7}{10}\right)^n + \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$$

$$= \frac{7}{3} + 9$$

$$= \frac{7+27}{3} = \frac{34}{3}$$

$$\sum_{n=1}^{\infty} \left(\frac{7}{10}\right)^n = \sum_{n=0}^{\infty} \left(\frac{7}{10}\right)^n - a_0$$

$$= \frac{1}{1-\frac{7}{10}} - 1 = \frac{1}{\frac{3}{10}} - 1 = \frac{10}{3} - 1$$

$$= \frac{10}{3} - \frac{3}{3}$$

$$= \frac{7}{3}$$

$$\sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n = \sum_{n=0}^{\infty} \left(\frac{9}{10}\right)^n - b_0$$

$$= \frac{1}{1-\frac{9}{10}} - 1 = \frac{1}{\frac{1}{10}} - 1 = 10 - 1$$

$$= 9$$

Question 3. *(5 marks)* §9.1 #35
Find the sum of the convergent series.

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \sum_{n=2}^{\infty} \frac{1}{(n+1)(n-1)} = \sum_{n=2}^{\infty} \left[\frac{-\frac{1}{2}}{n+1} + \frac{\frac{1}{2}}{n-1} \right]$$

$$\frac{1}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1}$$

$$1 = A(n-1) + B(n+1)$$

Let $n = 1$

$$1 = A(1-1) + B(1+1)$$

$$\frac{1}{2} = B$$

Let $n = -1$

$$1 = A(-1-1) + B(-1+1)$$

$$1 = -2A$$

$$\frac{1}{-2} = A$$

$$= \frac{1}{2} \sum_{n=2}^{\infty} \left[\frac{1}{n-1} - \frac{1}{n+1} \right]$$

$$= \frac{1}{2}\left[\frac{3}{2}\right]$$

$$= \frac{3}{4}$$

$$S_n = a_2 + a_3 + a_4 + a_5 + a_6 + \dots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \cancel{\frac{1}{3}}\right] + \left[\frac{1}{2} - \cancel{\frac{1}{4}}\right] + \left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right] + \left[\cancel{\frac{1}{4}} - \frac{1}{6}\right]$$

$$+ \left[\cancel{\frac{1}{5}} - \frac{1}{7}\right] + \dots + \left[\frac{1}{n-5} - \cancel{\frac{1}{n-3}}\right] + \left[\frac{1}{n-4} - \cancel{\frac{1}{n-2}}\right]$$

$$+ \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-1}}\right] + \left[\cancel{\frac{1}{n-2}} - \frac{1}{n}\right] + \left[\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right]$$

$$= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[1 + \frac{1}{2} - \overset{0}{\cancel{\frac{1}{n}}} - \overset{0}{\cancel{\frac{1}{n+1}}}\right] = \frac{3}{2}$$