Dawson College: **Calculus II**: 201-NYB-05-C2: Fall 2009

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Quiz 11

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §9.4 #23
Use the limit comparison test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

Let $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ be the test series, it converges since it is a p-series where $p = 2 > 1$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\cancel{n}\sqrt{n^2+1}} \cdot \frac{\cancel{n}\sqrt{n^2}}{1} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+1}} = 1$$

Since positive and finite behaves like the test series hence converges.

Question 2. *(5 marks)* §9.6 # 23
Use the ratio test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n!}{n\,3^n}$$

Let $a_n = \frac{n!}{n\,3^n}$

then
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)\,3^{n+1}} \cdot \frac{n\,3^n}{n!} \right|$$

$$= \lim_{n\to\infty} \frac{\cancel{(n+1)}\cancel{n!}\,n\,\cancel{3^n}}{\cancel{(n+1)}\cancel{3^n}\,3\,\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{n}{3} \to \infty > 1$$

$\therefore$ the series diverges by the ratio test.