Dawson College: Calculus II: 201-NYB-05-C2: Fall 2009

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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.6 #8 *(10 marks)*
Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for $n = 4$. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

$$\int_0^2 x\sqrt{x^2+1}\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

$$x_i = a + i\Delta x = \frac{i}{2}, \quad x_0 = \frac{1(0)}{2} = 0, \quad x_1 = \frac{1}{2}, \quad x_2 = \frac{2}{2} = 1, \quad x_3 = \frac{3}{2}, \quad x_4 = 2$$

Trapezoidal Rule:

$$\int_0^2 x\sqrt{x^2+1}\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{1}{4}\left[0\sqrt{0^2+1} + \cancel{(2)}\frac{1}{\cancel{2}}\sqrt{\left(\tfrac{1}{2}\right)^2+1} + (2)\sqrt{1^2+1} + \cancel{(2)}\frac{3}{\cancel{2}}\sqrt{\left(\tfrac{3}{2}\right)^2+1} + 2\sqrt{2^2+1}\right]$$

$$= \frac{1}{4}\left[\sqrt{\frac{5}{4}} + 2\sqrt{2} + 3\sqrt{\frac{13}{4}} + 2\sqrt{5}\right] \doteq 3.4567$$

Simpson's Rule:

$$\int_0^2 x\sqrt{x^2+1}\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{1}{6}\left[0\sqrt{0^2+1} + \overset{2}{\cancel{4}}\cdot\left(\frac{1}{\cancel{2}}\right)\sqrt{\left(\tfrac{1}{2}\right)^2+1} + 2(1)\sqrt{1^2+1} + \overset{2}{\cancel{4}}\left(\frac{3}{\cancel{2}}\right)\sqrt{\left(\tfrac{3}{2}\right)^2+1} + 2\sqrt{2^2+1}\right]$$

$$= \frac{1}{6}\left[2\sqrt{\frac{5}{4}} + 2\sqrt{2} + 6\sqrt{\frac{13}{4}} + 2\sqrt{5}\right] \doteq 3.3922$$

Exact Solution:

$$\int_0^2 x\sqrt{x^2+1}\,dx \overset{①,②}{=} \int_1^5 \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int_1^5 \sqrt{u}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_1^5 = \frac{5^{3/2}}{3} - \frac{1}{3} = \frac{\sqrt{125}-1}{3} \doteq 3.3934$$

$u \overset{①}{=} x^2+1$

$du = 2x\,dx$

$dx \overset{②}{=} \frac{du}{2x}$

$u(0) = 1$

$u(2) = 5$