Dawson College: **Calculus II**: 201-NYB-05-C2: Fall 2009

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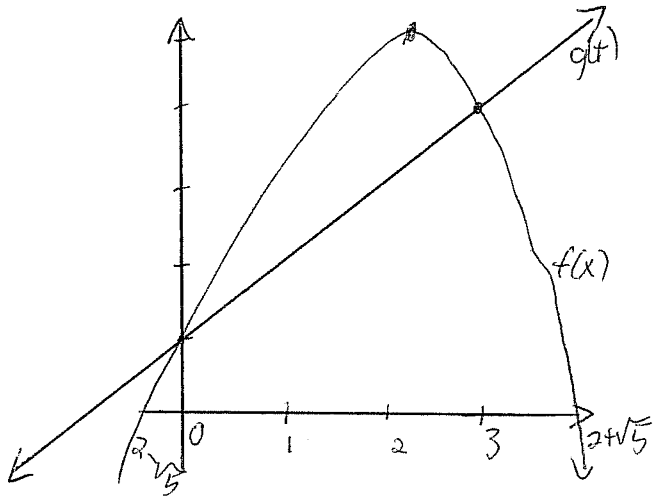
Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.1 #20 *(5 marks)*
Sketch the region bounded by the graphs of the algebraic functions

$f(x) = -x^2 + 4x + 1$ and $g(x) = x + 1$

and find the area of the region.



Lets find the intersection

$f(x) = g(x)$

$-x^2+4x+1 = x+1$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x=0 \quad x=3$

Sketch $g(x)$: x-int $(-1,0)$
y-int $(0,1)$

Sketch $f(x)$: orientation: $a=-1<0$ ∩

y-int $(0,1)$

x-int $0=f(x)$

$0 = -x^2+4x+1$

$0 = x^2-4x-1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2-4(1)(-1)}}{2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5} \doteq 4.2$ and -0.25

vertex $f(x) = -x^2+4x+1$

$= -[x^2-4x-1]$

$= -[(x^2-4x+4)-4-1]$

$= -[(x-2)^2-5]$

$= -(x-2)^2+5$

$\therefore$ vertex $(2,5)$

$$\text{Area} = \int_0^3 -x^2+4x+1-[x+1]\,dx$$

$$= \int_0^3 -x^2+3x\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3$$

$$= -\frac{\cancel{3^3}^{\,3^2}}{\cancel{3}} + \frac{3\,3^2}{2}$$

$$= -9 + \frac{27}{2}$$

$$= \frac{9}{2}$$

Question 2. §7.2 #32 *(5 marks)*
Find the volume of the solid generated by

$$y = 9 - x^2, y = 0, x = 2, x = 3$$

about the y-axis.

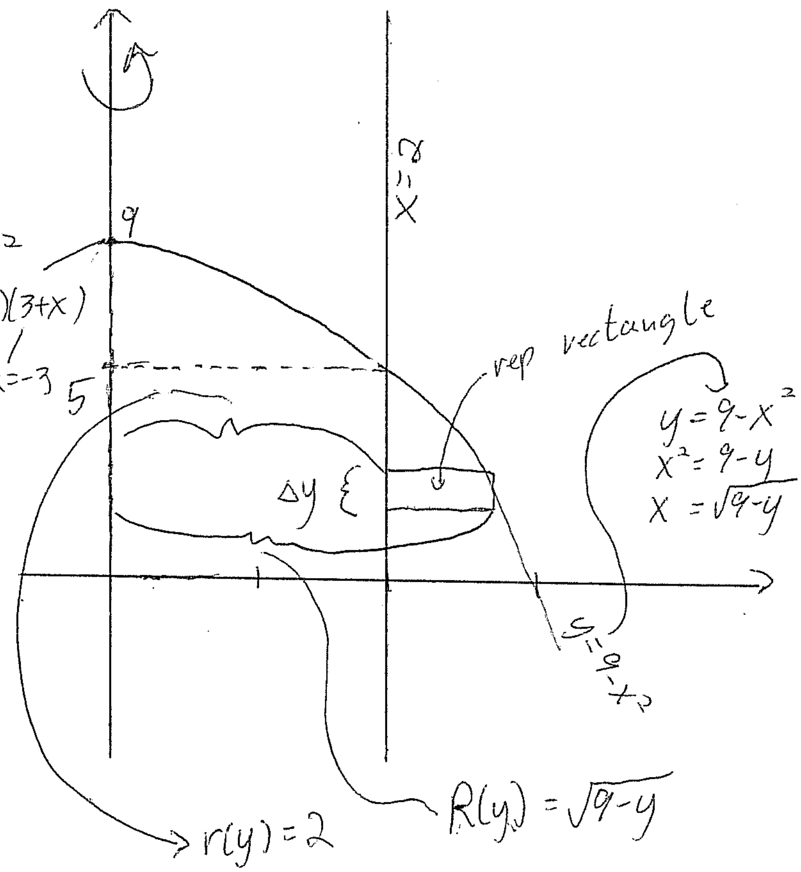
Sketch $y = 9 - x^2$: y-int. $(0, 9)$

x-int: $0 = 9 - x^2$

$0 = (3-x)(3+x)$

$x = 3 \quad x = -3$

vertex: $(0, 9)$



Representative element: $\Delta V = \pi[(R(y))^2 - (r(y))^2]\Delta y$

$= \pi[(\sqrt{9-y})^2 - (2)^2]\Delta y$

$= \pi[9 - y - 4]\Delta y$

$= \pi[5 - y]\Delta y$

$\therefore$ Volume $= \int_0^5 \pi[5 - y]dy$

$= \pi \int_0^5 5 - y\,dy$

$= \pi\left[5y - \frac{y^2}{2}\right]_0^5$

$= \pi\left[25 - \frac{25}{2}\right]$

$= \frac{25\pi}{2}$