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# Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(5 marks)* §8.2 #33
Integrate the following indefinite integral.

$\int \arctan x\, dx = uv - \int v\, du$

$= x\arctan x - \int \frac{x}{1+x^2} dx$

$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

$u = \arctan x$
$du = \frac{1}{1+x^2} dx$
$dv = dx$
$v = x$
u-substitution

**Question 2.** *(5 marks)* §8.4 #11
Integrate the following indefinite integral.

$\int x^3\sqrt{x^2-4}\, dx$

$x \overset{①}{=} 2\sec\theta$
$dx \overset{②}{=} 2\sec\theta\tan\theta\, d\theta$

$\overset{①,②}{=} \int (2\sec\theta)^3\sqrt{(2\sec\theta)^2-4}\cdot 2\sec\theta\tan\theta\, d\theta$

$= 8\int \sec^3\theta\sqrt{4\sec^2\theta-4}\, 2\sec\theta\tan\theta\, d\theta$

$= 8\int \sec^3\theta\sqrt{4(\sec^2\theta-1)}\, 2\sec\theta\tan\theta\, d\theta$

$= 16\int \sec^3\theta\sqrt{4\tan^2\theta}\,\sec\theta\tan\theta\, d\theta$

$= 32\int \sec^3\theta\tan\theta\sec\theta\tan\theta\, d\theta$

$= 32\int \tan^2\theta\sec^4\theta\, d\theta$

$= 32\int \tan^2\theta\sec^2\theta\sec^2\theta\, d\theta$

$= 32\int \tan^2\theta(1+\tan^2\theta)\sec^2\theta\, d\theta$

$u \overset{③}{=} \tan\theta$

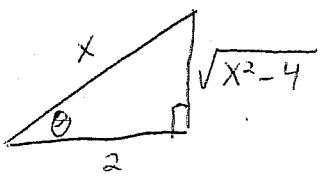
$du = \sec^2\theta\, d\theta$

$\frac{du}{\sec^2\theta} \overset{④}{=} d\theta$

From ①

$x = 2\sec\theta$

$\frac{hyp}{adj} = \frac{x}{2} = \sec\theta$ $\quad \cos\theta = \frac{adj}{hyp}$




$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{x^2-4}}{2}$

$= 32\int u^2(1+u^2)\sec^3\theta \frac{du}{\sec^3\theta}$ ②,④

$= 32\int u^2 + u^4\, du$

$= 32\left[\frac{u^3}{3} + \frac{u^5}{5}\right] + C$

$\overset{③}{=} 32\left[\frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5}\right] + C$

$= 32\left[\frac{\left(\frac{\sqrt{x^2-4}}{2}\right)^3}{3} + \frac{\left(\frac{\sqrt{x^2-4}}{2}\right)^5}{5}\right] + C$

$= 32\left[\frac{\left(\sqrt{x^2-4}\right)^3}{8\cdot 3} + \frac{\left(\sqrt{x^2-4}\right)^5}{32\cdot 5}\right] + C$

$= \frac{4\left(\sqrt{x^2-4}\right)^3}{3} + \frac{\left(\sqrt{x^2-4}\right)^5}{5} + C$