Dawson College: **Calculus II**: 201-NYB-05-C2: Fall 2009

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Quiz 9

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.8 #29
Determine if the improper integral diverges or converges. Evaluate if the integral converges.

$$\int_0^\infty \frac{1}{e^x + e^{-x}}\,dx$$

$$= \int_0^\infty \frac{1}{e^x + \frac{1}{e^x}}\,dx$$

$$= \int_0^\infty \frac{1}{\frac{e^{2x}+1}{e^x}}\,dx$$

$$= \int_0^\infty \frac{e^x}{e^{2x}+1}\,dx$$

$$= \lim_{b\to\infty} \int_0^b \frac{e^x}{(e^x)^2+1}\,dx$$

$u(x) \overset{①}{=} e^x \quad u(0) = e^0 = 1$

$du = e^x\,dx \quad u(b) = e^b$

$\frac{du}{e^x} \overset{②}{=} dx$

$$\overset{①②}{=} \lim_{b\to\infty} \int_1^{e^b} \frac{\cancel{e^x}}{u^2+1}\,\frac{du}{\cancel{e^x}}$$

$$= \lim_{b\to\infty} \int_1^{e^b} \frac{1}{u^2+1}\,du$$

$$= \lim_{b\to\infty} \Big[\arctan u\Big]_1^{e^b}$$

$$= \lim_{b\to\infty} \Big[\underbrace{\arctan e^b}_{\to \frac{\pi}{2}} - \arctan 1\Big]$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Question 2. *(5 marks)* §8.8 #35
Determine if the improper integral diverges or converges. Evaluate if the integral converges.

$$\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx$$

$$= \lim_{b\to 8^-} \int_0^b \frac{1}{\sqrt[3]{8-x}}\,dx \overset{①②}{=} \lim_{b\to 8^-} \int_8^{8-b} \frac{1}{\sqrt[3]{u}}\,(-du)$$

$u \overset{①}{=} 8-x$

$du = -dx$

$u(0) = 8-0 = 8$

$u(b) = 8-b$

$-du \overset{②}{=} dx$

$$= -\lim_{b\to 8^-} \int_8^{8-b} u^{-\frac{1}{3}}\,du$$

$$= -\lim_{b\to 8^-} \left[\frac{3u^{2/3}}{2}\right]_8^{8-b}$$

$$= -\lim_{b\to 8^-} \left[\underbrace{\frac{3}{2}(8-b)^{2/3}}_{\to 0} - \frac{3}{2}8^{2/3}\right]$$

$$= \frac{3}{2}8^{2/3} = \frac{3}{\cancel{2}}\cancel{4}^2 = 6$$

Question 3. *(5 marks)* §9.1 #64
Determine the convergence or divergence of the sequence with the given n^{th} term. If the sequence converges, find its limit.

$$a_n = n \sin \frac{1}{n}$$

$$\lim_{n \to \infty} n \sin \frac{1}{n} \qquad \text{IF.} \quad \infty \cdot 0$$

$$= \lim_{n \to \infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} \qquad \text{IF} \quad \frac{0}{0}$$

$$= \lim_{n \to \infty} \frac{\cos \frac{1}{n} \left(\cancel{\frac{-1}{n^2}} \right)}{\cancel{\frac{-1}{n^2}}} \qquad \text{by } \hat{H}$$

$$= 1$$