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Bonus Quiz 1

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.3 #20 *(5 marks)*
Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis

$$y = \sqrt{x+2}, \; y = x, \; y = 0.$$

Lets find the intersection of the two curves

$$x = \sqrt{x+2}$$
$$x^2 = x+2$$
$$0 = x^2 - x - 2$$
$$0 = (x-2)(x+1)$$
$$x = 2 \qquad \cancel{x = -1}$$
$$\text{not valid}$$

$$h(y) = y - [y^2 - 2]$$
$$= 2 + y - y^2$$

$$\therefore \; \Delta V = 2\pi p(y) h(y) \Delta y$$
$$= 2\pi (y)(2 + y - y^2) \Delta y$$

$$\text{Volume} = \int_0^2 2\pi (y)(2 + y - y^2) \, dy$$

$$= 2\pi \int_0^2 2y + y^2 - y^3 \, dy$$

$$= 2\pi \left[y^2 + \frac{y^3}{3} - \frac{y^4}{4} \right]_0^2$$

$$= 2\pi \left[2^2 + \frac{2^3}{3} - \frac{2^4}{4} \right] = \frac{16\pi}{3}$$

Question 2. §7.4 #7 *(5 marks)*
Find the arc length of the graph of the function

$$y = \frac{x^5}{10} + \frac{1}{6x^3}$$

over the interval $[1,2]$.

$$S = \int_1^2 \sqrt{1 + (y')^2}\, dx$$

$$= \int_1^2 \sqrt{\left(\frac{x^4}{2} + \frac{1}{2x^4}\right)^2}\, dx$$

$$= \int_1^2 \frac{x^4}{2} + \frac{1}{2x^4}\, dx$$

$$= \left[\frac{x^5}{10} - \frac{1}{6x^3}\right]_1^2$$

$$= \frac{2^5}{10} - \frac{1}{6 \cdot 2^3} - \left[\frac{1}{10} - \frac{1}{6}\right]$$

$$= \frac{32}{10} - \frac{1}{48} - \frac{1}{10} + \frac{1}{6}$$

$$= \frac{779}{240}$$

$$y' = \frac{5x^4}{10} - \frac{3}{6x^4}$$

$$= \frac{x^4}{2} - \frac{1}{2x^4}$$

$$\therefore \quad 1 + \left(\frac{x^4}{2} - \frac{1}{2x^4}\right)^2$$

$$= 1 + \frac{x^8}{4} - \frac{1}{2} + \frac{1}{4x^8}$$

$$= \frac{x^8}{4} + \frac{1}{2} + \frac{1}{4x^8}$$

$$= \left(\frac{x^4}{2} + \frac{1}{2x^4}\right)^2$$