

Name: _____
Student ID: _____

Test 2

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) Integrate the following indefinite integral:

$$\int \cos^2 2\theta \, d\theta$$

Question 2. (5 marks) Evaluate the following definite integral:

$$\int_0^{\frac{1}{2\sqrt{2}}} \arcsin(2x) \, dx$$

Question 3. (5 marks) A tank has the shape of a hemisphere with its round side up. The tank is 4 meters across the bottom. How much work is done in emptying the tank by pumping the water 5 m over the top? ($\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ and $g = 9.8 \frac{\text{m}}{\text{s}^2}$)

Question 4. (5 marks) Find the arclength of the graph of the function

$$y = \ln(\sin x)$$

over the interval $[\frac{\pi}{4}, \frac{3\pi}{4}]$.

Question 5. (5 marks) Sketch the region bounded by the graphs of the functions

$$f(x) = \sqrt{x-1} + 2 \text{ and } g(x) = x + 1$$

and find the area of the region.

Question 6. (5 marks) Set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region defined by:

$$f(x) = x^2 - 4x + 3, g(x) = -x^2 + 2x + 3$$

about $x = -1$.

Question 7. (5 marks) Set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region defined by:

$$f(x) = \sqrt{x+2} + 1, g(x) = 1, x = 2$$

about the x -axis.

Question 8. (5 marks) Integrate the following indefinite integral:

$$\int \frac{4x^2 + 2x - 1}{x^3 + x^2} dx$$

Question 9.(5 marks) Integrate the following indefinite integral:

$$\int x^3 \sqrt{2+x^2} dx$$

Bonus Question. (3 marks) Integrate the following indefinite integral:

$$\int x \arcsin x \, dx$$