

Name: _____
Student ID: _____

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Evaluate the following limit.

a. (5 marks)

$$\lim_{x \rightarrow \infty} x \arcsin \left(\frac{1}{x} \right)$$

b. (5 marks)

$$\lim_{x \rightarrow \infty} (1+x)^{1/x}$$

Question 2. Integrate the following improper integral.

a. (5 marks)

$$\int_1^7 \frac{2}{(1-x)^{5/3}} dx$$

b. (5 marks)

$$\int_1^\infty \frac{\arctan x}{1+x^2} dx$$

Question 3. Find the sum of the following series if it converges, if it diverges justify.

a. (5 marks)

$$\sum_{n=1}^{\infty} \frac{3^{n+1} + 2^n}{7^{n+1}}$$

b. (5 marks)

$$\sum_{n=10}^{\infty} \frac{2n^3 - n + 1}{3n^3 + 2n^2 - 1}$$

Question 4. Determine the convergence or divergence of the series.

a. (5 marks)

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n! 3^n}$$

b. (5 marks)

$$\sum_{n=1}^{\infty} \frac{e^{-\ln x}}{n}$$

Question 5. (5 marks) Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^5 - n^2 + 1}}$$

Question 6. (5 marks) Find the 2^{nd} degree Taylor polynomial of the function $f(x) = x^2 e^{2-2x} + \cos(2-2x)$ centered at $x = 1$.

Bonus Question. Let $f(x) = e^{3-3x}$

- a. (1 mark) Find a_n the n^{th} term of the Taylor polynomial of the function $f(x)$ centered at $x = 1$.
- b. (2 marks) For which value of x does the Taylor series

$$\sum_{n=0}^{\infty} a_n$$

converge.