Find the arc length of the graph of the function over the indicated interval.

$$S = \int_a^b \sqrt{1 + (y')^2}$$

$$y = \ln\left(\frac{e^x+1}{e^x-1}\right) \quad \text{on } [\ln 2, \ln 3]$$

$$y' = \frac{1}{\left[\frac{e^x+1}{e^x-1}\right]}\left[\frac{e^x(e^x-1) - e^x(e^x+1)}{(e^x-1)^2}\right]$$

$$= \frac{\cancel{e^x-1}}{e^x+1}\left[\frac{e^x[e^x-1-e^x-1]}{(e^x-1)^{\cancel{2}}}\right]$$

$$= \frac{-2e^x}{(e^x+1)(e^x-1)} = \frac{-2e^x}{e^{2x}-1}$$

$$\therefore S = \int_{\ln 2}^{\ln 3} \sqrt{1 + \left(\frac{-2e^x}{e^{2x}-1}\right)^2}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \sqrt{1 + \frac{4e^{2x}}{(e^{2x}-1)^2}}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \sqrt{\frac{(e^{2x}-1)^2 + 4e^{2x}}{(e^{2x}-1)^2}}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \sqrt{\frac{e^{4x} - 2e^{2x} + 1 + 4e^{2x}}{(e^{2x}-1)^2}}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \frac{\sqrt{e^{4x} + 2e^{2x} + 1}}{(e^{2x}-1)}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \frac{\sqrt{(e^{2x}+1)^2}}{(e^{2x}-1)}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \frac{e^{2x}+1}{e^{2x}-1}\,dx$$

add and remove

$$= \int_{\ln 2}^{\ln 3} \frac{e^{2x} + e^{2x} - e^{2x} + 1}{e^{2x}-1}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \frac{-e^{2x} + 1 + 2e^{2x}}{e^{2x}-1}\,dx$$

$$= \int_{\ln 2}^{\ln 3} \frac{-e^{2x}+1}{e^{2x}-1}\,dx + \int_{\ln 2}^{\ln 3} \frac{2e^{2x}}{e^{2x}-1}\,dx$$

$$= -\int_{\ln 2}^{\ln 3} \frac{\cancel{e^{2x}-1}}{\cancel{e^{2x}-1}}\,dx + \int_{\ln 2}^{\ln 3} \frac{2e^{2x}}{e^{2x}-1}\,dx$$

$$= -\left[x\right]_{\ln 2}^{\ln 3} + \left[\ln|e^{2x}-1|\right]_{\ln 2}^{\ln 3}$$

$$= -\ln 3 + \ln 2 + \left[\ln|e^{2\ln 3}-1| - \ln|e^{2\ln 2}-1|\right]$$

$$= \ln\frac{2}{3} + \ln|e^{\ln 3^2}-1| - \ln|e^{\ln 2^2}-1|$$

$$= \ln\frac{2}{3} + \ln|9-1| - \ln|4-1|$$

$$= \ln\frac{2}{3} + \ln 8 - \ln 3$$

$$= \ln\frac{2}{3} + \ln\frac{8}{3}$$

$$= \ln\frac{2}{3}\cdot\frac{8}{3}$$

$$= \ln\frac{16}{9}$$