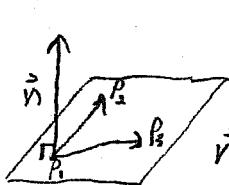


Bonus Assignment 1

This assignment is graded out of 9 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Find the equation of the plane passing through the points: $P_1(1, 2, 3)$, $P_2(-2, 0, -1)$, $P_3(0, -1, 0)$.



$$\vec{P_1P_2} = P_2 - P_1 = (-2, 0, -1) - (1, 2, 3) = (-3, -2, -4)$$

$$\vec{P_1P_3} = P_3 - P_1 = (0, -1, 0) - (1, 2, 3) = (-1, -3, -3)$$

$$\vec{n} = \vec{P_1P_2} \times \vec{P_1P_3} = \begin{vmatrix} -3 & -1 \\ -2 & -3 \end{vmatrix}, \begin{vmatrix} -3 & -1 \\ -1 & -3 \end{vmatrix}, \begin{vmatrix} -2 & -3 \\ -1 & -3 \end{vmatrix} = (-6, -5, -7)$$

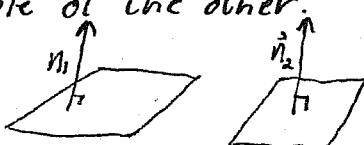
$$\begin{matrix} -3 & -1 \\ -2 & -3 \\ -4 & -3 \end{matrix} \quad \begin{matrix} -6x - 5y - 7z = d \\ -6(0) - 5(-1) - 7(0) = d \\ 5 = d \end{matrix} \quad \therefore -6x - 5y - 7z = 5$$

Question 2. Determine if the two planes are parallel: $2x + y - z = 101$ and $x + 2y - z = -101$.

2 planes are \parallel iff their normals are a multiple of the other.

i.e. $\vec{n}_1 = (2, 1, -1)$, $\vec{n}_2 = (1, 2, -1)$ $\vec{n}_1 \neq k\vec{n}_2$

\therefore not parallel

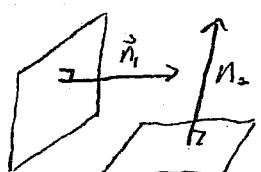


Question 3 Determine if the two planes are perpendicular: $2x - z = 101$ and $2y = -101$.

2 planes are \perp iff their normals are perpendicular

i.e. $\vec{n}_1 = (2, 0, -1)$, $\vec{n}_2 = (0, 2, 0)$ $\vec{n}_1 \cdot \vec{n}_2 = 0$

\therefore two planes are perpendicular

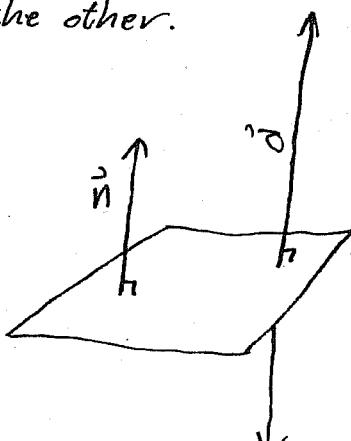


Question 4. Determine if the line and the plane are perpendicular: $(x, y, z) = (1, 2, 2) + t(2, -1, 2)$ and $-4x + 2y - 4z = 103$.

a line and a plane are perpendicular if the normal and the direction vector are a multiple of the other.

i.e. $\vec{n} = (-4, 2, -4)$ $\vec{d} = (2, -1, 2)$ $\vec{n} = -2\vec{d}$

\therefore the line and the plane are perpendicular

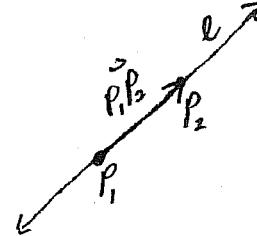


Question 6 Find the equation of the line passing through the given points: $P_1(8, -3, 4)$ and $P_2(2, 1, 2)$.

$$\ell = P_1 + tP_1P_2 \quad \text{where } \vec{P_1P_2} = P_2 - P_1$$

$$= (2, 1, 2) - (8, -3, 4)$$

$$= (-6, 4, -2)$$



Question 7 Find the equation for the line of intersection of the given planes: $-x + 2y - z = 3$ and $2x - y + 3z = 4$.

Find the line of intersection by solving the system.

$$\left[\begin{array}{cccc} -1 & 2 & -1 & 3 \\ 2 & -1 & 3 & 4 \end{array} \right] \rightarrow \left[\begin{array}{cccc} -1 & 2 & -1 & 3 \\ 0 & 3 & 1 & 10 \end{array} \right] \quad \left[\begin{array}{cccc} 1 & 0 & 5/3 & 11/3 \\ 0 & 1 & 1/3 & 10/3 \end{array} \right]$$

$$\sim 2R_1 + R_2 \rightarrow R_2 \quad \left[\begin{array}{cccc} -1 & 2 & -1 & 3 \\ 0 & 3 & 1 & 10 \end{array} \right] \quad z=t \text{ since } z \text{ is the free variable}$$

$$\sim 3R_1 \quad \left[\begin{array}{cccc} -3 & 6 & -3 & 9 \\ 0 & 3 & 1 & 10 \end{array} \right] \quad x = \frac{11}{3} - \frac{5}{3}t$$

$$\sim -2R_2 + R_1 \rightarrow R_1 \quad \left[\begin{array}{cccc} -3 & 0 & -5 & -11 \\ 0 & 3 & 1 & 10 \end{array} \right] \quad y = \frac{10}{3} - \frac{1}{3}t$$

Question 7. Find the equation of the plane that passes through the point $(2, -3, 4)$ and is parallel to the plane $2x - y - z - 301 = 0$. Two planes are parallel iff their normal are a multiple of each other.

$$\therefore 2x - y - z = d \quad \text{sub } (2, -3, 4) \quad 2(2) - (-3) - 4 = d$$

$$3 = d$$

$$\therefore 2x - y - z = 3$$

Question 8. Find an equation for the plane through $(3, -4, 3)$ that is perpendicular to the line of intersection of the planes $2x - y + 5z = 1$ and $-3x + y + 4z = 3$.

Find the line of intersection

$$\left[\begin{array}{cccc} 2 & -1 & 5 & 1 \\ -3 & 1 & 4 & 3 \end{array} \right] \sim R_2 \left[\begin{array}{cccc} 2 & -1 & 5 & 1 \\ 0 & 1 & -23 & -9 \end{array} \right] \quad z=t \text{ since } z \text{ is a free variable}$$

$$\sim 2R_2 \left[\begin{array}{cccc} 2 & -1 & 5 & 1 \\ -6 & 2 & 8 & 6 \end{array} \right] \quad x = -4 + 9t$$

$$\sim 3R_1 + R_2 \rightarrow R_1 \left[\begin{array}{cccc} 2 & 0 & -18 & -8 \\ 0 & 1 & -23 & -9 \end{array} \right] \quad y = -9 + 23t$$

$$\sim \frac{1}{2}R_1 \left[\begin{array}{cccc} 1 & 0 & -9 & -4 \\ 0 & 1 & -23 & -9 \end{array} \right] \quad \therefore d = (9, 23, 1)$$

$$\sim R_1 \left[\begin{array}{cccc} 1 & 0 & -9 & -4 \\ 0 & 1 & -23 & -9 \end{array} \right] \quad \text{which is normal of the plane}$$

$$9x + 23y + z = d$$

$$9(3) + 23(-4) + 3 = d$$

$$-62 = d$$

$$\therefore 9x + 23y + z = -62$$

Question 9. Find the intersection of the line $(x, y, z) = (1, 2, 1) + t(-2, 0, 1)$ and the plane $x - y + 3z = 6$.

$$\ell = \begin{cases} x = 1 - 2t \\ y = 2 \\ z = 1 + t \end{cases}$$

sub into eqn.

$$(1 - 2t) - (2) + 3(1 + t) = 6$$

$$1 - 2t - 2 + 3 + 3t = 6$$

$$t = 4$$

\therefore intersection at $t = 4$

$$x = 1 - 2(4)$$

$$x = -7$$

$$y = 2$$

$$y = 2$$

$$z = 1 + 4$$

$$z = 5$$

\therefore intersection at $(-7, 2, 5)$

