Dawson College: Linear Algebra: 201-105-DW-S04: Fall 2009

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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.*(5 marks)* Consider the matrix:

$$B = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

Find the matrix A if

$$(B^t - I + A^t)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\left[(B^t - I + A^t)^{-1}\right]^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1}$$

$$B^t - I + A^t = \frac{1}{4 - 2(3)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^t = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} + I - B^t$$

$$A^t = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -3 & -3 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} -3 & -3 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}^t$$

$$A = \begin{bmatrix} -3 & -\frac{3}{2} \\ -3 & \frac{1}{2} \end{bmatrix}$$

Question 2. *(5 marks)* Solve the following system by Gaussian elimination or Gauss-Jordan elimination.

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= -2 \\ 4x_1 + x_2 - 3x_3 &= 5 \\ 10x_1 - x_2 + x_3 &= 1 \end{aligned}$$

$$\begin{bmatrix} 3 & -1 & 2 & -2 \\ 4 & 1 & -3 & 5 \\ 10 & -1 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{matrix} \\ 3R_2 \\ 3R_3 \end{matrix} \begin{bmatrix} 3 & -1 & 2 & -2 \\ 12 & 3 & -9 & 15 \\ 30 & -3 & 3 & 3 \end{bmatrix}$$

$$\sim \begin{matrix} \\ -4R_1 + R_2 \to R_2 \\ -10R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 3 & -1 & 2 & -2 \\ 0 & 7 & -17 & 23 \\ 0 & 7 & -17 & 23 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \\ -R_2 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 3 & -1 & 2 & -2 \\ 0 & 7 & -17 & 23 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} 7R_1 \\ \\ \\ \end{matrix} \begin{bmatrix} 21 & -7 & 14 & -14 \\ 0 & 7 & -17 & 23 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} R_2 + R_1 \to R_1 \\ \\ \\ \end{matrix} \begin{bmatrix} 21 & 0 & -3 & 9 \\ 0 & 7 & -17 & 23 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} \frac{1}{21}R_1 \\ \frac{1}{7}R_2 \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & -3/21 & 9/21 \\ 0 & 1 & -17/7 & 23/21 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 is a free variable

hence $x_3 = t$ ③

$$\begin{cases} x_1 - \frac{3}{21}x_3 = \frac{9}{21} \\ x_2 - \frac{17}{7}x_3 = 23/21 \end{cases}$$

sub ③

$$\begin{cases} x_1 - \frac{1}{7}t = \frac{3}{7} \\ x_2 - \frac{17}{7}t = \frac{23}{21} \end{cases}$$

$$\begin{cases} x_1 = \frac{3}{7} + \frac{t}{7} \\ x_2 = \frac{23}{21} + \frac{17}{7}t \end{cases}$$

∴ the solution is

$x_1 = \frac{3}{7} + \frac{t}{7}$

$x_2 = \frac{23}{21} + \frac{17}{7}t$

$x_3 = t$

where $t \in \mathbb{R}$