Dawson College: **Linear Algebra**: 201-105-DW-S04: Fall 2009

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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.*(5 marks)* Consider the matrix:

$$B = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

Find the matrix A if

$$(B^t + I + A^{-1})^t = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\left[(B^t + I + A^{-1})^t\right]^t = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}^t$$

$$B^t + I + A^{-1} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} - I - B^t$$

$$A^{-1} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$$

$$(A^{-1})^{-1} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}^{-1}$$

$$A = \frac{1}{3(-1)-(-1)(1)} \begin{bmatrix} -1 & 1 \\ -1 & 3 \end{bmatrix}$$

$$A = \frac{1}{-2} \begin{bmatrix} -1 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

Question 2.*(5 marks)* Solve the following system by Gaussian elimination or Gauss-Jordan elimination.

$$\begin{aligned} 2x_1 &+ x_2 - 2x_3 = 2 \\ 5x_1 &- x_2 + 3x_3 = 2 \\ 9x_1 &+ x_2 - x_3 = 6 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & -2 & 2 \\ 5 & -1 & 3 & 2 \\ 9 & 1 & -1 & 6 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ 2R_2 \\ 2R_3 \end{array} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 10 & -2 & 6 & 4 \\ 18 & 2 & -2 & 12 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ -5R_1 + R_2 \to R_2 \\ -9R_1 + R_3 \to R_3 \end{array} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 0 & -7 & 16 & -6 \\ 0 & -7 & -16 & -6 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ \\ -R_2 + R_3 \to R_3 \end{array} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 0 & -7 & 16 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{c} 7R_1 \\ \\ \\ \end{array} \begin{bmatrix} 14 & 7 & -14 & 14 \\ 0 & -7 & 16 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{c} R_2 + R_1 \to R_1 \\ \\ \\ \end{array} \begin{bmatrix} 14 & 0 & 2 & 8 \\ 0 & -7 & 16 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{c} \frac{1}{14}R_1 \\ \frac{1}{-7}R_2 \\ \\ \end{array} \begin{bmatrix} 1 & 0 & \frac{1}{7} & \frac{4}{7} \\ 0 & 1 & \frac{-16}{7} & \frac{6}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} X_1 + \frac{1}{7}X_2 = \frac{4}{7} \\ X_2 - \frac{16}{7}X_3 = \frac{6}{7} \end{cases}$$

X_3 is a free variable hence

$X_3 = t$ (A)

soln (B)

$$\begin{cases} X_1 + \frac{1}{7}t = \frac{4}{7} \\ X_2 - \frac{16}{7}t = \frac{6}{7} \end{cases}$$

$$\begin{cases} X_1 = \frac{4}{7} - \frac{t}{7} \\ X_2 = \frac{6}{7} + \frac{16t}{7} \end{cases}$$

$\therefore$ the solution is

$X_1 = \frac{4}{7} - \frac{t}{7}$

$X_2 = \frac{6}{7} + \frac{16t}{7}$

$X_3 = t$

where $t \in \mathbb{R}$