**Dawson College: Linear Algebra**: 201-105-DW-S04: Fall 2009

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# Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Consider the matrix:

$$B=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & \sqrt{2}\end{bmatrix}, C=\begin{bmatrix}1 & 0 & 2 & 1\\ 2 & 0 & 3 & 2\\ -2 & -1 & 0 & -2\\ -2 & 0 & 0 & 0\end{bmatrix}$$

a. *(3 marks)* Compute $\det(B^{-2})$

b. *(4 marks)* Compute $\det(C)$

c. *(3 marks)* If A is an $n \times n$ symmetric matrix then show that $2A^2 - 3A + I$ is symmetric.

a) $B^{-2} = \begin{bmatrix}(\frac{1}{5})^{-2} & 0 & 0\\ 0 & 3^{-2} & 0\\ 0 & 0 & (\sqrt{2})^{-2}\end{bmatrix} = \begin{bmatrix}4 & 0 & 0\\ 0 & \frac{1}{9} & 0\\ 0 & 0 & \frac{1}{2}\end{bmatrix}$

$\det(B^{-2}) = 4(\frac{1}{9})(\frac{1}{2}) = \frac{2}{9}$

b) $\det C = c_{41}C_{41} + c_{42}C_{42} + c_{43}C_{43} + c_{44}C_{44}$

$= -2C_{41} + 0C_{42} + 0C_{43} + 0C_{44}$

$= -2(-1)^{4+1}\begin{vmatrix}0 & 2 & 1\\ 0 & 3 & 2\\ -1 & 0 & -2\end{vmatrix}$

$= -2(-1)\left[c_{11}C_{11} + c_{21}C_{21} + c_{31}C_{31}\right]$

$= -2(-1)\left[0C_{11} + 0C_{21} + (-1)\begin{vmatrix}2 & 1\\ 3 & 2\end{vmatrix}\right]$

$= -2(-1)(-1)[2(2) - 3(1)]$

$= -2(-1)(-1)(1)$

$= -2$

c) Lets show that $(2A^2 - 3A + I)^t = (2A^2 - 3A + I)$

$(2A^2 - 3A + I)^T = (2A^2)^T - (3A)^T + I^T$

$= 2(A^2)^T - 3A^T + I$ since $I^T = I$

$= 2(A^T)^2 - 3A^T + I$

$= 2A^2 - 3A + I$ since $A^T = A$