Dawson College: **Linear Algebra**: 201-105-DW-S04: Fall 2009

Name: Y. LAMONTAGNE
Student ID:

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 2 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ -2 & 0 & 5 & 0 \end{bmatrix}$$

$$B^{-2} = \begin{bmatrix} (2)^{-2} & 0 & 0 \\ 0 & (\frac{1}{3})^{-2} & 0 \\ 0 & 0 & (\sqrt{3})^{-2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

a. *(3 marks)* Compute $\det(B^{-2})$

$$\therefore \det(B^{-2}) = \frac{1}{4}(9)\left(\frac{1}{3}\right) = \frac{3}{4}$$

b. *(4 marks)* Compute $\det(C)$

c. *(3 marks)* If A is an $n \times n$ symmetric matrix then show that $2A^2 - 3A + I$ is symmetric.

b)
$$\det C = c_{12} C_{12} + c_{22} C_{22} + c_{32} C_{32} + c_{42} C_{42}$$
$$= 0\, C_{12} + 0\, C_{22} + (-1)(-1)^{3+2} \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \\ -2 & 5 & 0 \end{vmatrix} + 0\, C_{42}$$
$$= (-1)(-1)\left[c_{13} C_{13} + c_{23} C_{23} + c_{33} C_{33} \right]$$
$$= (-1)(-1)\left[1 (-1)^{1+3} \begin{vmatrix} -2 & 1 \\ -2 & 5 \end{vmatrix} + 0\, C_{23} + 0\, C_{33} \right]$$
$$= (-1)(-1)(1)(1)\left[-10 + 2\right]$$
$$= (-1)(-1)(1)(1)(-8)$$
$$= -8$$

c) Lets show that $(2A^2 - 3A + I)^T = 2A^2 - 3A + I$

$$(2A^2 - 3A + I)^T = (2A^2)^T - (3A)^T + I^T$$
$$= 2(A^2)^T - 3A^T + I \quad \text{since } I^T = I$$
$$= 2(A^T)^2 - 3A^T + I$$
$$= 2A^2 - 3A + I \quad \text{since } A^T = A$$