

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, B = \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & -2 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

a. (3 marks) If $\det(A) = -9$ and $\det(B) = \frac{1}{2}$ then compute $\det((2A)^T B^2 (3AB)^{-1})$.

b. (3 marks) If $\det(A) = -9$, compute

$$\det \begin{pmatrix} 2a & 2b & 2c \\ 4g & 4h & 4i \\ d+g & e+h & f+i \end{pmatrix}$$

c. (4 marks) Compute $\det(C)$.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\sim \begin{matrix} 2R_1 \\ R_2 \leftrightarrow R_3 \end{matrix} \begin{bmatrix} 2a & 2b & 2c \\ g & h & i \\ d & e & f \end{bmatrix}$$

$$\begin{aligned} (2)(-1)(4) \det A &= \det B \\ -8(-9) &= \det B \\ 72 &= \det B \end{aligned}$$

$$\sim \begin{matrix} 4R_2 \\ R_2 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2a & 2b & 2c \\ 4g & 4h & 4i \\ d+g & e+h & f+i \end{bmatrix} = B$$

$$\begin{aligned} 2 \left(\frac{1}{2}\right) \det C &= \det D \\ \det C &= 3 \end{aligned}$$

$$C \sim \begin{matrix} 2R_2 \\ \frac{1}{3}R_4 \end{matrix} \begin{bmatrix} 2 & 1 & -2 & 1 & 0 \\ 6 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{matrix} -3R_1 + R_2 \rightarrow R_2 \\ -R_4 + R_5 \rightarrow 0 \end{matrix} \begin{bmatrix} 2 & 1 & -2 & 1 & 0 \\ 0 & 1 & 8 & -3 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = D$$

$$\begin{aligned} &= \det(2A)^T \det B^2 \det(3AB)^{-1} \\ &= \det(2A) (\det B)^2 \frac{1}{\det(3AB)} \\ &= 2^3 \det A (\det B)^2 \frac{1}{3^3 \det A \det B} \\ &= \frac{2^3}{3^3} \det B \\ &= \frac{2^3}{3^3} \left(\frac{1}{2}\right) = \frac{4}{27} \end{aligned}$$