

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, B = \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}, C = \begin{bmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 3 & 3 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}$$

- a. (3 marks) If $\det(A) = -\frac{1}{3}$ and $\det(B) = 5$ then compute $\det((4AB)^{-1}(2A)^t B^3)$.
- b. (3 marks) If $\det(A) = -\frac{1}{3}$, compute

$$\det \begin{pmatrix} 3g & 3h & 3i \\ 4d & 4e & 4f \\ a+g & b+h & c+i \end{pmatrix}$$

- c. (4 marks) Compute $\det(C)$.

b)

$$A \sim R_1 \leftrightarrow R_3 \begin{bmatrix} g & h & i \\ d & e & f \\ a & b & c \end{bmatrix}$$

$$\sim \begin{bmatrix} g & h & i \\ d & e & f \\ a+g & b+h & c+i \end{bmatrix}$$

$R_1 + R_3 \rightarrow R_3$

$$\begin{aligned} (-1)(3)(4) \det A &= \det D \\ (-1)(3)(4) \frac{-1}{3} &= \det D \\ 4 &= \det D \end{aligned}$$

$$C \sim \begin{bmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 2 & 6 & 6 \\ 2 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$2R_4$
 $2R_5$

$$\begin{aligned} &= \det(4AB)^{-1} \det(2A)^t \det B^3 \\ &= \frac{1}{\det(4AB)} \det(2A) (\det B)^3 \\ &= \frac{1}{4^3 \det A \det B} 2^3 \det A (\det B)^3 \\ &= \frac{2^3 (\det B)^2}{4^3} = \frac{2^3 5^2}{4^3} \\ &= \frac{25}{8} \end{aligned}$$

$$\sim \begin{bmatrix} 3g & 3h & 3i \\ 4d & 4e & 4f \\ a+g & b+h & c+i \end{bmatrix} = D$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 0 & 8 & 5 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} = E$$

$-R_3 + R_4$
 $-R_1 + R_5$

$$\begin{aligned} (2)(2) \det C &= \det E \\ \det C &= \frac{1}{2} \frac{1}{2} \cdot 2 \cdot 2 \cdot 8 \cdot 2 \\ &= 32 \end{aligned}$$