

## Quiz 6

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Let  $\mathbf{u} = (2, 3, -2)$ ,  $\mathbf{a} = (3, -1, -1)$ .

a. (1 mark) Compute  $\mathbf{a} \cdot \mathbf{a}$ .  $\vec{a} \cdot \vec{a} = 9 + (-1)(-1) + (-1)(-1) = 11$

b. (1 mark) Compute  $\mathbf{u} \cdot \mathbf{a}$ .  $\vec{u} \cdot \vec{a} = 2(3) + 3(-1) + (-2)(-1) = 5$

c. (2 marks) Compute  $\text{proj}_{\mathbf{a}} \mathbf{u}$ .  $= \frac{\vec{u} \cdot \vec{a}}{\vec{a} \cdot \vec{a}} \vec{a} = \frac{5}{11} (3, -1, -1) = \left( \frac{15}{11}, -\frac{5}{11}, -\frac{5}{11} \right)$

d. (1 mark) Find the vector component of  $\mathbf{u}$  orthogonal to  $\mathbf{a}$ .  $\vec{u} - \text{proj}_{\vec{a}} \vec{u} = (2, 3, -2) - \left( \frac{15}{11}, -\frac{5}{11}, -\frac{5}{11} \right) = \left( \frac{7}{11}, \frac{38}{11}, -\frac{17}{11} \right)$

**Question 2.** (5 marks) Calculate the angle between the vector  $\mathbf{w} = (1, 8)$  and the line  $y = -3x + 5$ .

Let  $x=0 \Rightarrow y = -3(0) + 5 = 5$   $\therefore A(0, 5)$

$x=1 \Rightarrow y = -3(1) + 5 = 2$   $\therefore B(1, 2)$

$\vec{AB} = B - A = (1, 2) - (0, 5) = (1, -3)$  has same direction

as the line

$\therefore$  Lets find the angle between  $\vec{w}$  and  $\vec{AB}$

$$\|\vec{w}\| = \sqrt{1^2 + 8^2} = \sqrt{65}$$

$$\|\vec{AB}\| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$$

and  $\vec{w} \cdot \vec{AB} = (1)(1) + 8(-3) = -23$

$$\vec{w} \cdot \vec{AB} = \|\vec{w}\| \|\vec{AB}\| \cos \theta$$

$$\cos \theta = \frac{\vec{w} \cdot \vec{AB}}{\|\vec{w}\| \|\vec{AB}\|}$$

$$\cos \theta = \frac{-23}{\sqrt{10} \sqrt{65}}$$

$$\theta = \cos^{-1} \left( \frac{-23}{\sqrt{10} \sqrt{65}} \right)$$

$$\theta \doteq 154.4^\circ \text{ or } 25.6^\circ$$