

Quiz 6

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $\mathbf{u} = (4, 1, -2)$, $\mathbf{a} = (-3, 1, 1)$.

a. (1 mark) Compute $\mathbf{a} \cdot \mathbf{a}$. $\vec{a} \cdot \vec{a} = (-3)(-3) + (1)(1) + (1)(1) = 11$

b. (1 mark) Compute $\mathbf{u} \cdot \mathbf{a}$. $\vec{u} \cdot \vec{a} = (4)(-3) + (1)(1) + (-2)(1) = -13$

c. (2 marks) Compute $\text{proj}_{\mathbf{a}} \mathbf{u}$. $= \frac{\vec{u} \cdot \vec{a}}{\vec{a} \cdot \vec{a}} \vec{a} = \frac{-13}{11} (-3, 1, 1) = \left(\frac{39}{11}, -\frac{13}{11}, -\frac{13}{11} \right)$

d. (1 mark) Find the vector component of \mathbf{u} orthogonal to \mathbf{a} . $\mathbf{u} - \text{proj}_{\mathbf{a}} \mathbf{u} = (4, 1, -2) - \left(\frac{39}{11}, -\frac{13}{11}, -\frac{13}{11} \right) = \left(\frac{5}{11}, \frac{24}{11}, -\frac{9}{11} \right)$

Question 2. (5 marks) Calculate the angle between the vector $\mathbf{w} = (2, -5)$ and the line $y = -4x + 2$.

Let $x=0 \Rightarrow y = -4(0) + 2 = 2$: A (0, 2)

Let $x=1 \Rightarrow y = -4(1) + 2 = -2$: B (1, -2)

$$\vec{AB} = B - A$$

$$= (1, -2) - (0, 2)$$

$$= (1, -4)$$

same direction as the line.

∴ Let's find the angle θ between \vec{w} and \vec{AB}

$$\|\vec{w}\| = \sqrt{(2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$$

$$\|\vec{AB}\| = \sqrt{1^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17}$$

and $\vec{w} \cdot \vec{AB} = (2, -5) \cdot (1, -4)$

$$= 2 + 20$$

$$= 22$$

$$\vec{w} \cdot \vec{AB} = \|\vec{w}\| \|\vec{AB}\| \cos \theta$$

$$\cos \theta = \frac{\vec{w} \cdot \vec{AB}}{\|\vec{w}\| \|\vec{AB}\|}$$

$$\cos \theta = \frac{22}{\sqrt{17} \sqrt{29}}$$

$$\theta = \cos^{-1} \left(\frac{22}{\sqrt{17} \sqrt{29}} \right)$$

$$\theta \approx 7.8 \text{ or } 172.2^\circ$$