Name: Y. Lamontagne
Student ID: _____

Quiz 7

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* Let $\mathbf{u} = (1,2,-1)$, $\mathbf{v} = (2,-2,-2)$, $\mathbf{w} = (2,-1,-3)$. Find the volume of the parallelepiped with sides **u**, **v**, **w**.

$$\vec{u} \cdot (\vec{v} \times \vec{w})$$

$$= \begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & -2 \\ 2 & -1 & -3 \end{vmatrix} = 1(-1)^{1+1}\begin{vmatrix} -2 & -2 \\ -1 & -3 \end{vmatrix} + 2(-1)^{2+1}\begin{vmatrix} 2 & -2 \\ 2 & -3 \end{vmatrix} + (-1)(-1)^{3+1}\begin{vmatrix} 2 & -2 \\ 2 & -1 \end{vmatrix}$$

$$= [6-2] + 2[-6+4] - [-2+4]$$

$$= 6$$

∴ volume of the parallelpiped is 6

Question 2.

a. *(3 marks)* Find a vector that is orthogonal to both $\mathbf{x} = (1,-2,2)$ and $\mathbf{y} = (-2,2,-3)$.

b. *(1 marks)* Find the area of the parallelogram determined by **x** and **y**.

$$\vec{x} \times \vec{y} = \left(\begin{vmatrix} -2 & 2 \\ 2 & -3 \end{vmatrix}, -\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix}, \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix}\right) = (2,-1,-2)$$

$$\begin{matrix} 1 & -2 \\ -2 & 2 \\ 2 & -3 \end{matrix}$$

$$\text{area} = \|\vec{x} \times \vec{y}\| = \|(2,-1,-2)\| = \sqrt{2^2+(-1)^2+(-2)^2} = \sqrt{9} = 3$$

Question 3. *(2 marks)* Find a vector **v** that is orthogonal to the vector $\mathbf{u} = (2,-5,3)$.

Let $\vec{v} = (x_1, x_2, x_3)$

$$0 = \vec{v} \cdot \vec{u} = (x_1, x_2, x_3) \cdot (2, -5, 3)$$

$$0 = 2x_1 - 5x_2 + 3x_3$$

Let $x_1 = 1$, $x_2 = 2$

$$0 = 2(1) - 5(2) + 3x_3$$

$$8 = 3x_3$$

$$\frac{8}{3} = x_3$$

∴ $(1, 2, \frac{8}{3}) = \vec{v}$