

Quiz 7

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $\mathbf{u} = (3, 2, -1)$, $\mathbf{v} = (3, -3, -3)$, $\mathbf{w} = (2, -2, -3)$.

a. (3 marks) Find the scalar triple product of \mathbf{u} , \mathbf{v} , \mathbf{w} .

b. (1 mark) Find the volume of the parallelepiped with sides \mathbf{u} , \mathbf{v} , \mathbf{w} .

$$\begin{aligned} a) \quad \vec{u} \cdot (\vec{v} \times \vec{w}) &= \begin{vmatrix} 3 & 2 & -1 \\ 3 & -3 & -3 \\ 2 & -2 & -3 \end{vmatrix} = 3(-1)^{1+1} \begin{vmatrix} -3 & -3 \\ -2 & -3 \end{vmatrix} + 2(-1)^{2+1} \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} + -1(-1)^{3+1} \begin{vmatrix} 3 & -3 \\ 2 & -2 \end{vmatrix} \\ &= 3[9 - 6] + 2(-1)[-9 + 6] \\ &= 9 + 6 = 15 \end{aligned}$$

b) the volume is 15.

Question 2. (4 marks) Find the area of the parallelogram determined by $\mathbf{x} = (2, -3, 3)$ and $\mathbf{y} = (-2, 2, -3)$.

$$\vec{x} \times \vec{y} = \left(\begin{vmatrix} -3 & 2 \\ 3 & -3 \end{vmatrix}, - \begin{vmatrix} 2 & -2 \\ 3 & -3 \end{vmatrix}, \begin{vmatrix} 2 & -2 \\ -3 & 2 \end{vmatrix} \right) = (3, 0, -2)$$

$$\text{area} = \|\vec{x} \times \vec{y}\| = \sqrt{3^2 + 0^2 + (-2)^2} = \sqrt{13}$$

Question 3. (2 marks) Find a vector \mathbf{v} that is orthogonal to the vector $\mathbf{u} = (1, -6, -2)$.

$$\text{Let } \vec{v} = (x_1, x_2, x_3)$$

$$\begin{aligned} 0 &= \vec{v} \cdot \vec{u} = (x_1, x_2, x_3) \cdot (1, -6, -2) \\ 0 &= x_1 - 6x_2 - 2x_3 \end{aligned} \quad \therefore (1, 1, \frac{-5}{2})$$

$$\text{Let } x_1 = 1, x_2 = 1$$

$$\begin{aligned} 0 &= 1 - 6(1) - 2x_3 \\ 2x_3 &= -5 \\ x_3 &= \frac{-5}{2} \end{aligned}$$