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Quiz 7

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* Let $\mathbf{u} = (2,3,-2)$, $\mathbf{v} = (3,-1,-1)$, $\mathbf{w} = (2,-1,-3)$. Find the volume of the parallelepiped with sides **u**, **v**, **w**.

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} 2 & 3 & -2 \\ 3 & -1 & -1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= 2(-1)^{1+1}\begin{vmatrix} -1 & -1 \\ -1 & -3 \end{vmatrix} + 3(-1)^{1+2}\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} + (-2)(-1)^{1+3}\begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix}$$

$$= 2[3-1] - 3[-9+2] - 2[-3+2]$$

$$= 4 + 21 + 2 = 27$$

∴ the volume of the parallelepiped is 27.

Question 2.

a. *(3 marks)* Find a vector that is orthogonal to both $\mathbf{x} = (3,-2,1)$ and $\mathbf{y} = (-1,2,-3)$.

b. *(1 marks)* Find the area of the parallelogram determined by **x** and **y**.

$$\vec{x}\times\vec{y} = \left(\begin{vmatrix} -2 & 2 \\ 1 & -3 \end{vmatrix}, -\begin{vmatrix} 3 & -1 \\ 1 & -3 \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix}\right) = (4, 8, 4)$$

$$\begin{matrix} 3 & -1 \\ -2 & 2 \\ 1 & -3 \end{matrix}$$

∴ the area $= \|\vec{x}\times\vec{y}\| = \|(4,8,4)\| = \sqrt{4^2+8^2+4^2}$

$$= \sqrt{96}$$

Question 3. *(2 marks)* Suppose that $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = 17$ then find $(\mathbf{w}\times\mathbf{v})\cdot\mathbf{u}$ and justify.

$$(\vec{w}\times\vec{v})\cdot\vec{u} = \vec{u}\cdot(\vec{w}\times\vec{v})$$

$$= -\vec{u}\cdot(\vec{v}\times\vec{w})$$

same as interchanging two rows of the determinant

$$= -17$$