EXAMPLE 10 Simplifying expression from calculus

$$\begin{aligned}
3(x+4)^2(x-3)^{-2} - 2(x-3)^{-3}(x+4)^3 &= \frac{3(x+4)^2}{(x-3)^2} - \frac{2(x+4)^3}{(x-3)^3} = \frac{3(x-3)(x+4)^2 - 2(x+4)^3}{(x-3)^3} \\
&= \frac{(x+4)^2[3(x-3) - 2(x+4)]}{(x-3)^3} \\
&= \frac{(x+4)^2(x-17)}{(x-3)^3}
\end{aligned}$$

Expressions like the one in this example are often found in problems in calculus.

EXERCISES 11.1

In Exercises 1–4, solve the resulting problems if the given changes are made in the indicated examples of this section.

1. In Example 3, change the factor x^2 to x^{-2} and then find the result.

2. In Example 6, change the term $2a$ to $2a^{-1}$ and then find the result.

3. In Example 8(b), change the 3^{-1} in the denominator to 3^{-2} and then find the result.

4. In Example 9, change the sign in the numerator from $-$ to $+$ and then find the result.

In Exercises 5–52, express each of the given expressions in simplest form with only positive exponents.

5. x^7x^{-4} **6.** y^9y^{-2} **7.** $2a^2a^{-6}$

8. $5ss^{-5}$ **9.** $5^0 \times 5^{-3}$ **10.** $(3^2 \times 4^{-3})^3$

11. $(2\pi x^{-1})^2$ **12.** $(3xy^{-2})^3$ **13.** $2(5an^{-2})^{-1}$

14. $4(6s^2t^{-1})^{-2}$ **15.** $(-4)^0$ **16.** -4^0

17. $-7x^0$ **18.** $(-7x)^0$ **19.** $3x^{-2}$

20. $(3x)^{-2}$ **21.** $(7a^{-1}x)^{-3}$ **22.** $7a^{-1}x^{-3}$

23. $\left(\frac{2}{n^3}\right)^{-3}$ **24.** $\left(\frac{3}{x^3}\right)^{-2}$ **25.** $3\left(\frac{a}{b^{-2}}\right)^{-3}$

26. $5\left(\frac{2n^{-2}}{D^{-1}}\right)^{-2}$ **27.** $(a+b)^{-1}$ **28.** $a^{-1} + b^{-1}$

29. $3x^{-2} + 2y^{-2}$ **30.** $(3x + 2y)^{-2}$

31. $(2a^{-n})^2\left(\frac{3}{2a^n}\right)^{-1}$ **32.** $(7 \times 3^{-a})\left(\frac{3^a}{7}\right)^2$

33. $\left(\frac{3a^2}{4b}\right)^{-3}\left(\frac{4}{a}\right)^{-5}$ **34.** $(2np^{-2})^{-2}(4^{-1}p^2)^{-1}$

35. $\left(\frac{V^{-1}}{2t}\right)^{-2}\left(\frac{t^2}{V^{-2}}\right)^{-3}$ **36.** $ab\left(\frac{a^{-2}}{b^2}\right)^{-3}\left(\frac{a^{-3}}{b^5}\right)^2$

37. $2a^{-2} + (2a^{-2})^4$ **38.** $3(a^{-1}z^2)^{-3} + c^{-2}z^{-1}$

39. $2 \times 3^{-1} + 4 \times 3^{-2}$ **40.** $5 \times 2^{-2} - 3^{-1} \times 2^3$

41. $(R_1^{-1} + R_2^{-1})^{-1}$ **42.** $2(2a - b^{-2})^{-1}$ **43.** $(n^{-2} - 2n^{-1})^2$

44. $2(2^{-3} - 4^{-1})^{-2}$ **45.** $\frac{6^{-1}}{4^{-2} + 2}$ **46.** $\frac{x - y^{-1}}{x^{-1} - y}$

47. $\frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}}$ **48.** $\frac{ax^{-2} + a^{-2}x}{a^{-1} + x^{-1}}$ **49.** $2t^{-2} + t^{-1}(t + 1)$

50. $3x^{-1} - x^{-3}(y + 2)$ **51.** $(D - 1)^{-1} + (D + 1)^{-1}$

52. $4(2x - 1)(x + 2)^{-1} - (2x - 1)^2(x + 2)^{-2}$

In Exercises 53–70, solve the given problems.

53. If $x < 0$, is it ever true that $x^{-2} < x^{-1}$?

54. Is it true that $(a + b)^0 = 1$ for all values of a and b?

55. Express $4^2 \times 64$ (a) as a power of 4 and (b) as a power of 2.

56. Express 1/81 (a) as a power of 9 and (b) as a power of 3.

57. (a) By use of Eqs. (11.4) and (11.6), show that

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

(b) Verify the equation in part (a) by evaluating each side with $a = 3.576$, $b = 8.091$, and $n = 7$.

(W) **58.** For what integral values of n is $(-3)^{-n} = -3^{-n}$? Explain.

59. For what integral value(s) of n is $n^\pi > \pi^n$?

(W) **60.** Evaluate $(8^{19})^{12}/(8^{16})^{14}$. What happens when you try to evaluate this on a calculator?

61. Solve for x: $2^{5x} = 2^7(2^{2x})^2$.

62. In analyzing the tuning of an electronic circuit, the expression $[\omega\omega_0^{-1} - \omega_0\omega^{-1}]^2$ is used. Expand and simplify this expression.

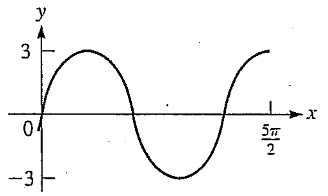
63. The metric unit of energy, the *joule* (J), can be expressed as $kg \cdot s^{-2} \cdot m^2$. Simplify these units and include *newtons* (see Appendix B) and only positive exponents in the final result.

64. The units for the electric quantity called *permittivity* are $C^2 \cdot N^{-1} \cdot m^{-2}$. Given that $1\ F = 1\ C^2 \cdot J^{-1}$, show that the units of permittivity are F/m. See Appendix B.

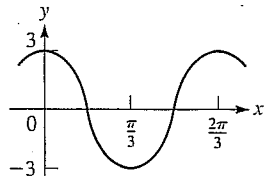
65. When studying a solar energy system, the units encountered are $kg \cdot s^{-1}(m \cdot s^{-2})^2$. Simplify these units and include *joules* (see Example 4) and only positive exponents in the final result.

53. 4π

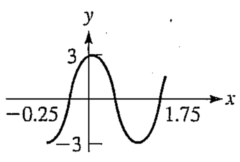
55. $y = 3\sin x$



57. $y = 3\cos 3x$



59. $y = 3\sin(\pi x + 0.25\pi)$



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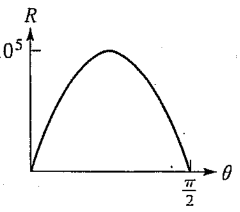

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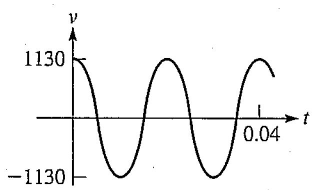

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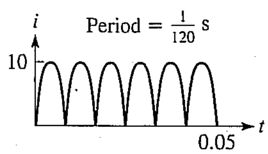

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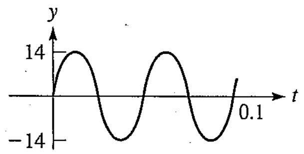

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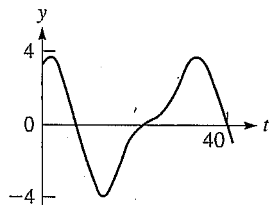

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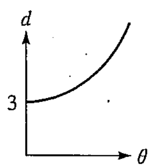

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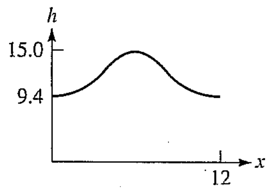

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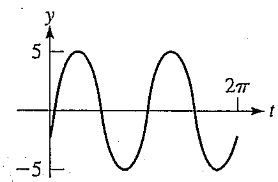

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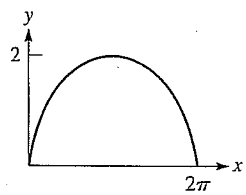

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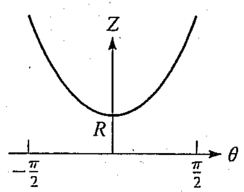

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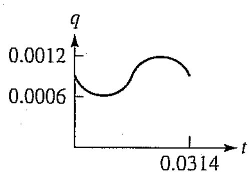


Exercises 11.1, page 316

1. $\frac{y^2}{4x^2}$ **3.** $\frac{3}{416}$ **5.** x^3 **7.** $\frac{2}{a^4}$ **9.** $\frac{1}{125}$ **11.** $\frac{4\pi^2}{x^2}$

13. $\frac{2n^2}{5a}$ **15.** 1 **17.** −7 **19.** $\frac{3}{x^2}$ **21.** $\frac{a^3}{343x^3}$

23. $\frac{n^9}{8}$ **25.** $\frac{3}{a^3b^6}$ **27.** $\frac{1}{a + b}$ **29.** $\frac{2x^2 + 3y^2}{x^2y^2}$

31. $\frac{8}{3a^n}$ **33.** $\frac{b^3}{432a}$ **35.** $\frac{4}{t^4V^4}$ **37.** $\frac{2a^6 + 16}{a^8}$ **39.** $\frac{10}{9}$

41. $\frac{R_1R_2}{R_1 + R_2}$ **43.** $\frac{4n^2 - 4n + 1}{n^4}$ **45.** $\frac{8}{99}$ **47.** $\frac{x + y}{xy}$

49. $\frac{t^2 + t + 2}{t^2}$ **51.** $\frac{2D}{D^2 - 1}$ **53.** No

55. (a) 4^5 (b) 2^{10}

57. (a) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$

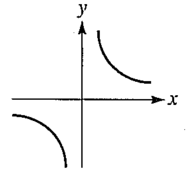
(b) 303.551 82 = 303.551 82

59. $n = 3$ **61.** 7 **63.** N · m **65.** J/s³ **67.** 1

69. $\frac{p\left[(1 + i)^n - 1\right]}{i(1 + i)^n}$

Exercises 11.2, page 320

1. 16 **3.**



5. 5 **7.** 3

9. 10^{25} **11.** $\frac{1}{2}$ **13.** $\frac{1}{16}$ **15.** 25 **17.** 81 **19.** −200

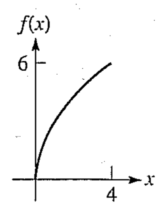
21. $\frac{3}{5}$ **23.** −2 **25.** $\frac{39}{1000}$ **27.** $\frac{3}{5}$ **29.** 2.059

31. 0.538 91 **33.** $B^{7/6}$ **35.** $\frac{1}{-y^{9/10}}$ **37.** $\frac{1}{x^{3/2}}$ **39.** $2ab^2$

41. $\frac{1}{8a^3b^{9/4}}$ **43.** $a^{1/12}$ **45.** $\frac{4x}{(4x^2 + 1)^{1/2}}$ **47.** $-y$

49. $\frac{T}{(T + 2)^{1/2}}$ **51.** $\frac{a^2 + 1}{a^4}$ **53.** $\frac{a + 1}{a^{1/2}}$ **55.** $\frac{5x^2 - 2x}{(2x - 1)^{1/2}}$

57.



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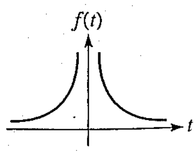

61. $\sqrt[3]{x^2}$

63. If $(A/S)^{-1/4} = 0.5 = 1/2$, then $(A/S)^{1/4} = 2$. Raise each to the fourth power and get $A/S = 16$.

65. $R = \frac{T^{2/3}}{k^{1/3}} - d$ **67.** 1.91 mA