

NAME: SOLUTIONS

TEST 1

Dawson College

Applied Math (201-943-DW S1)

Date: Sep 24th 2010

Instructor: E. Richer

Question 1. (10 marks)

Evaluate the following.

(a) $\sqrt[3]{-27} = \boxed{-3}$

(b) $-4^{-0.5} = -\frac{1}{4^{0.5}} = -\frac{1}{4^{1/2}} = \boxed{-\frac{1}{2}}$

(c) $9^{\frac{2}{3}} = \boxed{(\sqrt[3]{9})^2}$

(d) $\sqrt[4]{625} = \boxed{5}$

(e) $\sqrt[3]{-8} = \boxed{-2}$

Question 2. (6 marks)

Evaluate the following. Express your answer in the three following ways:

(i) Scientific notation (ii) Engineering notation (iii) Eng. not. with metric prefixes

(a) $0.00345 + 3.6 \times 10^{-3}$
 $= 0.00705$

(i) 7.05×10^{-3} (ii) 7.05×10^{-3} (iii) 7.05 m

(b) $64.5\text{M} + 12.2\text{k}$

$= 64.5 \times 10^6 + 12.2 \times 10^3$

$= 64512200$

(i) 6.45122×10^7 (ii) 64.5122×10^6 (iii) 64.5122M

(c) $\frac{3.26 \times 10^{-5} - 0.000027}{43 \times 10^{-7}}$

$= \frac{0.0000326 - 0.000027}{43 \times 10^{-7}}$

$= \frac{0.0000056}{43 \times 10^{-7}} = 1.3$

Question 3. (4 marks)

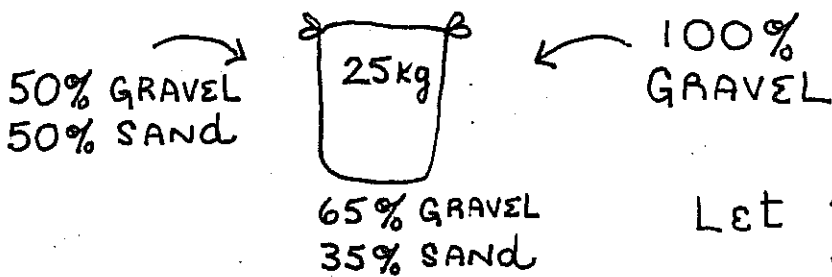
Define, in your own words, a **number in engineering notation**. Give the general form of a number expressed in engineering notation.

A number in engineering notation is expressed in the form

$$P \times 10^n \quad \text{WHERE } n \text{ is a multiple of 3} \\ \& \quad 1 \leq P < 1000$$

Question 4. (7 marks)

A mixture of 65% gravel and 35% sand must be used by the City of Montreal for de-icing roads. Bags of the mixture, weighing 25kg each, are to be distributed to city residents. To fill the bags the city must use its old stock, a mixture of 50% gravel-50% sand and add gravel. How much gravel must be added to the old mixture in order to fill one bag?



$$\text{Let } x = \text{Kg OF GRAVEL} \\ y = \text{Kg OF 50/50 MIX}$$

$$x + y = 25 \Rightarrow x = 25 - y$$

$$x + 0.5y = 0.65(25)$$

$$25 - y + 0.5y = 16.25$$

$$-0.5y = -8.75$$

$$y = 17.5 \text{ Kg}$$

$$x = 7.5 \text{ Kg}$$

Question 5. (4 marks)

Solve the system of equations.

$$2x - 3y = -4$$

$$x + 2y = 5 \rightarrow x = 5 - 2y$$

SUBSTITUTE IN EQUATION ①

$$2x - 3y = -4$$

$$2(5 - 2y) - 3y = -4$$

$$10 - 4y - 3y = -4$$

$$-7y = -14$$

$$y = 2$$

$$\rightarrow x = 5 - 2y \\ = 5 - 2(2) = 1$$

$$\text{SO } x = 1 \quad y = 2$$

Question 6. (9 marks)

Isolate the specified variable in the given expressions:

(a) $\frac{2}{x} + 4y = \frac{y}{5} - 3$ (isolate x)

MULTIPLY BOTH SIDES OF THE EQUATION BY x

$$2 + 4xy = \frac{xy}{5} - 3x$$

MULTIPLY BOTH SIDES BY 5

$$10 + 20xy = xy - 15x$$

$$10 + 19xy = -15x$$

$$19xy + 15x = -10$$

$$x(19y + 15) = -10 \rightarrow$$

$$x = \frac{-10}{19y + 15}$$

(b) $a^2b - 3(a+b) = \frac{3}{a} - 2b$ (isolate b)

$$a^2b - 3a - 3b = \frac{3}{a} - 2b \quad \text{MULTIPLY BOTH SIDES BY } a$$

$$a^3b - 3a^2 - 3ab = 3 - 2ab$$

$$a^3b - 3a^2 - ab = 3$$

$$b(a^3 - a) = 3 + 3a^2$$

$$b = \frac{3 + 3a^2}{a^3 - a}$$

(c) $x^{-1}y^2 - \frac{3y}{2} = 24$ (isolate x)

$$\frac{y^2}{x} - \frac{3y}{2} = 24 \quad \text{MULTIPLY BOTH SIDES BY } 2x$$

$$2y^2 - 3xy = 48x$$

$$2y^2 = 48x + 3xy$$

$$2y^2 = x(48 + 3y)$$

$$x = \frac{2y^2}{48 + 3y}$$

Question 7. (6 marks).

Solve the system of equations.

$$2x + 2y + z = 5$$

$$x + 2y - z = 4$$

$$x + y - 3z = -1$$

ISOLATE x IN EQU^N 3.

$$x = -1 - y + 3z$$

SUB IN EQU^NS 1. & 2.

$$1. \quad 2(-1 - y + 3z) + 2y + z = 5$$

$$-2 - 2y + 6z + 2y + z = 5$$

$$7z = 7$$

$$z = 1$$

$$2. \quad -1 - y + 3z + 2y - z = 4$$

$$y + 2z = 5$$

SUB IN $z = 1$

$$y + 2(1) = 5$$

$$y = 3$$

$$3. \quad \text{SUB IN } y = 3 \text{ \& } z = 1$$

$$x = -1 - y + 3z$$

$$x = -1 - (3) + 3(1)$$

$$= -1$$

$$\boxed{x = -1 \quad y = 3 \quad z = 1}$$

Question 8. (14 marks)

TRUE or FALSE: For each of the following statements, indicate whether it is true or false. If the statement is false, give a correction to the statement.

(a) $-5^{-2} = 25$

$-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$ **FALSE**

(b) $(-125)^{\frac{2}{3}} = 25$

$(-125)^{\frac{2}{3}} = (\sqrt[3]{-125})^2 = 25$ **TRUE**

(c) $\frac{4x^3}{2x+8} = 2x^2 + \frac{x^3}{2}$

$\frac{4x^3}{2x+8}$

$\frac{2x^2-8x+32}{2x+8} \begin{array}{r} 4x^3 \\ -(4x^3+16x^2) \end{array}$ **FALSE**

$= \frac{2x^2-8x+32-256}{2x+8}$

$\frac{-16x^2}{-(-16x^2-64x)}$

(d) $\frac{1}{x^{-1}} = x$

$\frac{1}{x^{-1}} = \frac{1}{x}$

FALSE

$\frac{64x}{-(64x+256)}$
 -256

(e) 0.24×10^{-6} is expressed in engineering notation

FALSE

240×10^{-9}

(f) $2 - x(3+x) = -6x - 2x^2$

$2 - x(3+x)$
 $= 2 - 3x - x^2$

FALSE

(g) $-2^{-3} = (-2)^{-3}$

TRUE

(h) $\sqrt[4]{-16} = -2$

FALSE

$\sqrt[4]{-16}$ does not exist \mathbb{R}

Question 9. (16 marks)

Perform the following operations on polynomials and simplify the answer.

(a) $4a^5 - 2a + 3(a - 2) - 5a^4(a + 2)$

$$= 4a^5 - 2a + 3a - 6 - 5a^5 - 10a^4$$

$$= \boxed{-a^5 - 10a^4 + a - 6}$$

(b) $2y(y - 1)(5 - 3y)$

$$= (2y^2 - 2y)(5 - 3y)$$

$$= 10y^2 - 6y^3 - 10y + 6y^2$$

$$= \boxed{-6y^3 + 16y^2 - 10y}$$

(c) $\frac{2xy - 4x^2y}{xy} = \frac{2xy}{xy} - \frac{4x^2y}{xy}$

$$= \boxed{2 - 4x}$$

(d) $2xy^2 - x(x - 3(4 - y))$

$$= 2xy^2 - x(x - 12 + 3y)$$

$$= \boxed{2xy^2 - x^2 + 12x - 3xy}$$

(e) $(x^3 - 4x + 3) \div (x - 1)$

$$= \boxed{x^2 + x - 3}$$

$$\begin{array}{r} x-1 \overline{) x^3 - 4x + 3} \\ \underline{-(x^3 - x^2)} \\ x^2 - 4x + 3 \\ \underline{-(x^2 - x)} \\ -3x + 3 \\ \underline{-(-3x + 3)} \\ 0 \end{array}$$

(Question 10.) (5 marks)

Sean is hiking to an isolated fishing cabin, his trip involves a hike along a river followed by a canoe trip. The distance travelled on foot is the same distance as travelled by boat. Sean walks at a speed of 4 km/h and paddles at a speed of 10 km/hr. The total trip from the parking lot to the fishing cabin takes him 5 hours and 30 minutes. How many kilometers did he travel in total?

$$\begin{aligned} \text{distance on FOOT} &= d \\ \text{" " by BOAT} &= d \text{ (SAME)} \end{aligned}$$

$$\begin{aligned} \text{distance on FOOT} &= (\text{speed on FOOT}) \cdot (\text{time spent on foot}) \\ \text{distance by boat} &= (\text{speed by boat}) \cdot (\text{time spent on boat}) \end{aligned}$$

$$\text{Let } x = \text{time walking}$$

$$y = \text{time by boat}$$

$$x + y = 5 \text{ h } 30 \text{ min} = 5.5 \text{ h}$$

$$y = 5.5 - x$$

$$d = x(4)$$

$$d = y(10) = (5.5 - x)10$$

$$\text{so } 4x = (5.5 - x)10$$

$$4x = 55 - 10x$$

$$14x = 55$$

$$x = 55/14 \text{ so } d = 55/14 (4) = 15.71 \text{ KM}$$

(Question 11.) (4 marks) Simplify the following expression.

TOTAL $15.71 \text{ KM} \times 2$

31.42 KM

$$\frac{-2^{-3}x^{-4}y^2}{\frac{2^2x^3y^{-5}}{(2^{-1}x^2y^2)^{-2}}} = (-2^{-3}x^{-4}y^2) \cdot \frac{(2^{-1}x^2y^2)^{-2}}{2^2x^3y^{-5}}$$

$$= (-2^{-3}x^{-4}y^2) \cdot \frac{2^2x^{-4}y^{-4}}{2^2x^3y^{-5}}$$

$$= -2^{-3}x^{-4}y^2 \cdot \frac{y}{x^7}$$

$$= -\frac{y^3}{2^3x^{11}}$$

$$= \boxed{-\frac{y^3}{8x^{11}}}$$

Question 12. (15 marks)

Using the laws of exponents, simplify the following expressions. There should be no negative exponents in your final answer.

$$\begin{aligned} \text{(a)} & (-2^3(-x)^5)(2^4x^6) \\ &= (2^3x^5)(2^4x^6) \\ &= \boxed{2^7x^{11}} \end{aligned}$$

$$\text{(b)} \frac{5}{-125x^{-2}} = \frac{5x^2}{-125} = \boxed{-\frac{x^2}{25}}$$

$$\text{(c)} \frac{2^3x^3y^{-3}}{(4x)^2y^{-3}} = \frac{2^3x^3}{4^2x^2} = \frac{8x}{16} = \boxed{\frac{x}{2}}$$

$$\begin{aligned} \text{(d)} & (2^{-1}a^3b^{-1})^{-4} \\ &= 2^4a^{-12}b^4 \\ &= \boxed{\frac{16b^4}{a^{12}}} \end{aligned}$$

$$\begin{aligned} \text{(e)} & (3^3x^{-3}y^{-2})^4(9x^2y^{-4}) \\ &= (3^{12}x^{-12}y^{-8})(9x^2y^{-4}) \\ &= 3^{14}x^{-10}y^{-12} \\ &= \boxed{\frac{3^{14}}{x^{10}y^{12}}} \end{aligned}$$