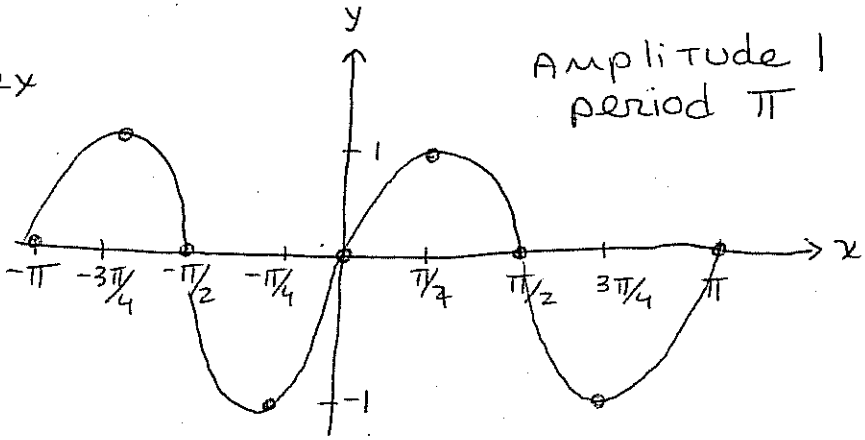
BONUS TRIG FUNCTIONS GRAPHING
943-DW

A- $y = \sin 2x$

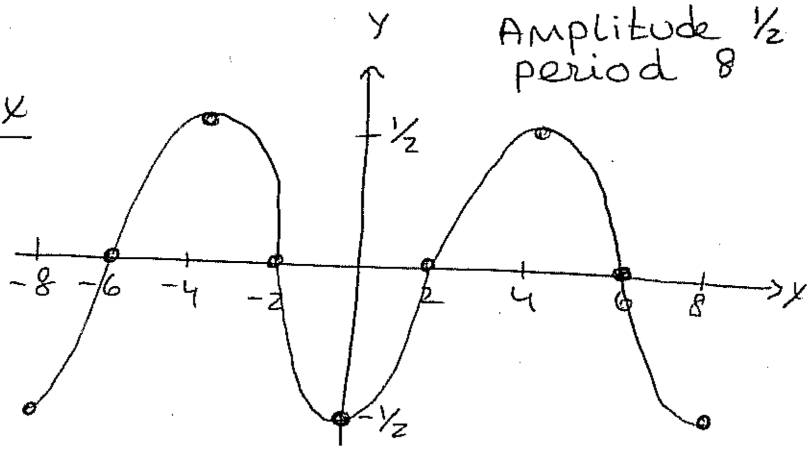
x	Angle $2x$	$y = \sin 2x$
0	0	0
$\pi/4$	$\pi/2$	1
$\pi/2$	π	0
$3\pi/4$	$3\pi/2$	-1
π	2π	0



Amplitude 1
period π

B- $y = -\frac{1}{2}\cos \frac{\pi}{4} x$

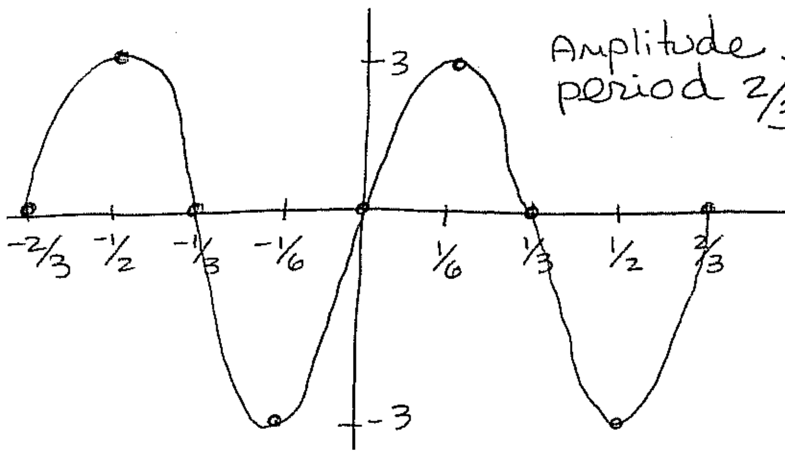
x	Angle $\frac{\pi}{4}x$	$y = -\frac{1}{2}\cos\frac{\pi}{4}x$
0	0	$-\frac{1}{2}$
2	$\pi/2$	0
4	π	$\frac{1}{2}$
6	$3\pi/2$	0
8	2π	$-\frac{1}{2}$



Amplitude $\frac{1}{2}$
period 8

C- $y = 3\sin(3\pi x)$

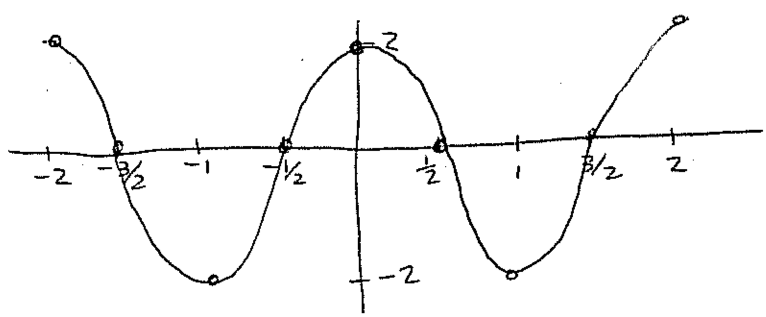
x	Angle $3\pi x$	$y = 3\sin 3\pi x$
0	0	0
$\frac{1}{6}$	$\pi/2$	3
$\frac{1}{3}$	π	0
$\frac{1}{2}$	$3\pi/2$	-3
$\frac{2}{3}$	2π	0



Amplitude 3
period $\frac{2}{3}$

D- $y = 2\cos(\pi x)$

x	Angle πx	$y = 2\cos\pi x$
0	0	2
$\frac{1}{2}$	$\pi/2$	0
1	π	-2
$\frac{3}{2}$	$3\pi/2$	0
2	2π	2



NAME: SOLUTIONS

TEST 3

Dawson College

Applied Math (201-943-DW S1)

Date: Dec 3rd 2010

Instructor: E. Richer

Question 1.

Solve the following equations.

a. *(4 marks)*

$\log_3(x-5)+\log_3(x+4)=2$

$$\log_3 (x-5)(x+4) = 2$$

$$(x-5)(x+4) = 3^2$$

$$x^2 - x - 20 = 9$$

$$x^2 - x - 29 = 0$$

$$x = \frac{1 \pm \sqrt{1-4(-29)}}{2}$$

$$= \frac{1 \pm \sqrt{117}}{2}$$

But ONLY $x = \frac{1+\sqrt{117}}{2}$ is a solution

b. *(4 marks)*

$3^{x-2}=4$

$$(x-2)\ln 3 = \ln 4$$

$$x-2 = \frac{\ln 4}{\ln 3}$$

$$\boxed{x = \frac{\ln 4}{\ln 3} + 2}$$

Question 2. *(3 marks each)*

Perform the following operations involving complex numbers. Express your final answer in rectangular form $a+bj$.

a. $j^3+\sqrt{-4}-j^2(3+j)$

$= -j + 2j - (-1)(3+j)$

$= -j + 2j + 3 + j$

$= \boxed{3+2j}$

b. $\frac{2-j}{4+2j}$

$= \frac{2-j}{4+2j}\,\frac{(4-2j)}{(4-2j)}$

$= \frac{8-4j-4j+2j^2}{16-4j^2}$

$= \frac{6-8j}{20} = \boxed{\frac{3}{10}-\frac{2}{5}j}$

c. $\sqrt{-64}j+j^{63}-\frac{1+j}{j^5}$

$= 8j^2 + j^{60}\cdot j^3 - \frac{1+j}{j^4 j}$

$= -8 - j - \frac{1+j}{j}$

$= -8 - j - \left(\frac{1+j}{j}\right)\frac{j}{j}$

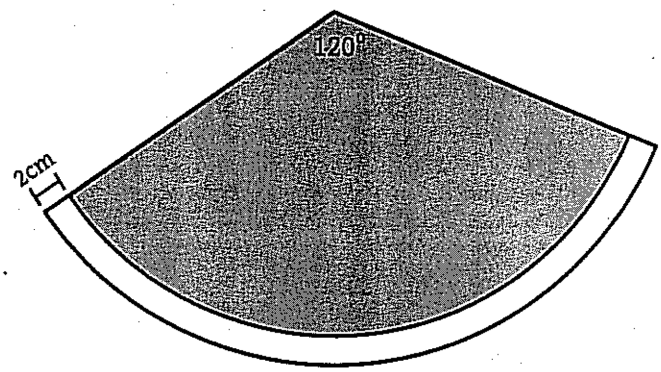
$= -8 - j - \left[\frac{j+j^2}{j^2}\right]$

$= -8 - j - \left[\frac{j-1}{-1}\right]$

$= -8 - j + j - 1$

$= \boxed{-9}$

Question 3. *(5 marks)*



The sector of a circle pictured above has an area of $48\pi \text{cm}^2$.

a. Find the radius of the "shaded" part of the sector.
b. Calculate the area of the area of the "shaded" part of the sector.

a. $A = \frac{1}{2} r^2 \theta$ $\quad$ θ in radians $\quad$ $120° = 120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ rad

$48\pi = \frac{1}{2} r^2 \left(\frac{2\pi}{3}\right)$

$48\pi = \frac{\pi}{3} r^2$

$144 = r^2$

$r = 12\text{cm}$ $\quad$ so radius of shaded part is $12\text{cm} - 2\text{cm} = \boxed{10\text{ cm}}$

b. $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(10)^2\left(\frac{2\pi}{3}\right)$

$= \boxed{\frac{100\pi}{3}\text{ cm}^2}$

Question 4. *(5 marks)*
Solve the following equation for θ, $0^\circ \leq \theta < 360^\circ$.
$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x = 1$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$x = 90^\circ, 270^\circ$ $\qquad$ $x = 0^\circ$

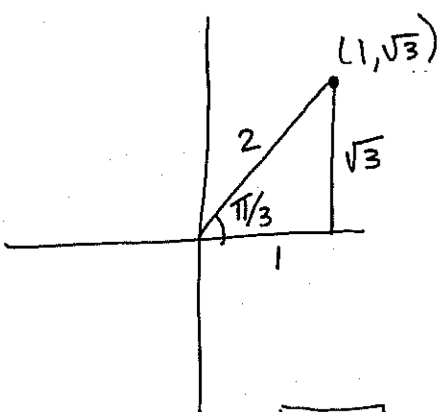
SOLUTIONS : $\boxed{x = 0^\circ, 90^\circ, 270^\circ}$

Question 5. *(2 marks each)*
Find the *exact* values of the following.

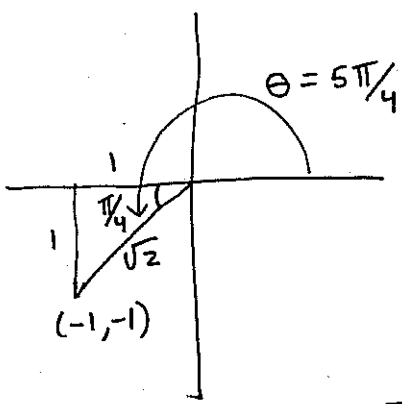
a. $\sin\frac{\pi}{3} = \boxed{\sqrt{3}/2}$

b. $\cos\frac{5\pi}{4} = \boxed{-1/\sqrt{2}}$

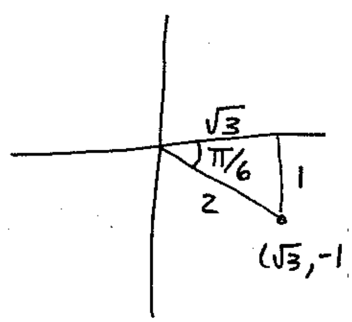
c. $\cot\frac{11\pi}{6} = \boxed{-\sqrt{3}}$



$\sin\frac{\pi}{3} = \frac{y}{r} = \boxed{\sqrt{3}/2}$

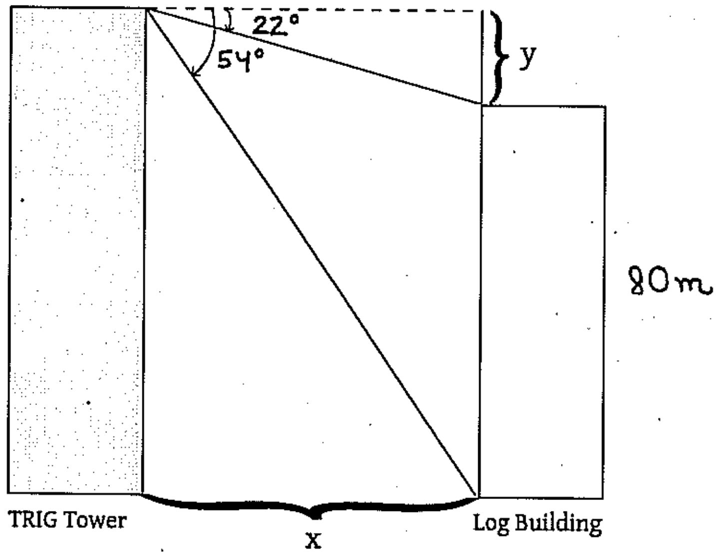


$\cos\frac{5\pi}{4} = x/r = \boxed{-1/\sqrt{2}}$



$\cot\pi/6 = \frac{x}{y} = \frac{\sqrt{3}}{-1}$
$= \boxed{-\sqrt{3}}$

Question 6. *(6 marks)*
The angle depression from the top of the TRIG Tower to the top of the shorter LOG Building across the street is 22° and the angle of depression from the top of TRIG Tower to the bottom of the LOG Building is 54°. If the Log Building is 80m in height, how far apart are the two buildings?



① $\tan 22° = \dfrac{Y}{x}$

$Y = x\tan 22°$

② $\tan 54° = \dfrac{80+Y}{x}$

$Y = x\tan 54° - 80$

COMBINING THE TWO EQUATIONS we get

$$x\tan 22° = x\tan 54° - 80$$

$$x(\tan 22° - \tan 54°) = -80$$

$$x = \frac{-80}{\tan 22° - \tan 54°} = \frac{-80}{0.404 - 1.376} = \frac{-80}{-0.972}$$

$$= \boxed{82.304\text{ m}}$$

Question 7. *(2 marks each)*
Using properties of logarithms, express each of the following as a single logarithm.

a. $\ln x - 3\ln x^2 + 5\ln 2x$

$$= \ln\left[\frac{x}{(x^2)^3}\right] + \ln(2x)^5$$

$$= \ln\left[\frac{x(32x^5)}{x^6}\right] = \boxed{\ln 32}$$

b. $(\log_3 4)(\log_3 x)$

$$= \boxed{\log_3\left(x^{\log_3 4}\right)}$$

Question 8. *(2 marks each)*
Evaluate each of the following.

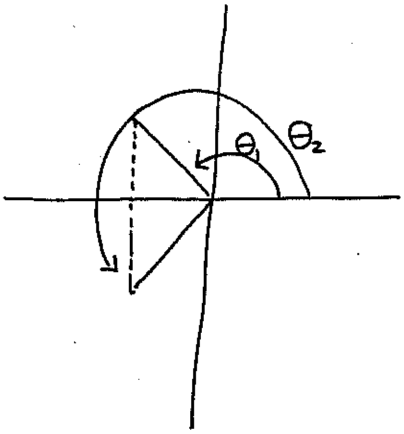
a. $\log_2 32$

$$= \log_2(2^5) = \boxed{5}$$

b. $\ln\frac{1}{e^2}$ $= \ln(e^{-2}) = \boxed{-2}$

c. $\frac{\log_3 27}{\log_5 25^{-1}}$ $= \dfrac{\log_3 3^3}{\log_5 5^{-2}} = \boxed{\dfrac{3}{-2}}$

Question 9. *(5 marks)*
Solve for x in the equation $\cos x = -0.35$ subject to the conditions $\sin x < 0$ and $0° \leq x < 360°$



$\cos^{-1}(-0.35) = 110.5°$

$\theta_1 = 110.5°$

$\theta_2 = 360° - 110.5°$
$= 249.5°$

SINCE $\sin x < 0$

THE ONLY SOLUTION IS $\boxed{x = 249.5°}$

Question 10. *(6 marks)*
Find the *three* cube roots of $-27j$. Express your answers in rectangular form.

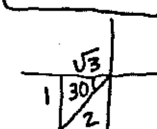
WE WANT $(-27j)^{1/3}$

$-27j$ IN EXPONENTIAL FORM IS ① $27e^{270°j}$
or ② $27e^{630°j}$ (Add 360°)
or ③ $27e^{990j}$ (Add 360°)

TAKING THE CUBE ROOTS WE HAVE:

① $(-27j)^{1/3} = (27e^{270j})^{1/3}$
$= 3e^{90j}$
$= 3\cos 90° + 3\sin 90° j$
$= \boxed{0 + 3j}$

② $(27e^{630j})^{1/3}$
$= 3e^{210°j}$
$= 3\cos 210° + 3\sin 210° j$
$= 3\left(-\frac{\sqrt{3}}{2}\right) + 3\left(-\frac{1}{2}\right)j$
$= \boxed{-\frac{3\sqrt{3}}{2} - \frac{3}{2}j}$



③ $(27e^{990j})^{1/3}$
$= 3e^{330j}$
$= 3\cos 330° + 3\sin 330$
$= \boxed{\frac{3\sqrt{3}}{2} - \frac{3}{2}j}$