DAWSON COLLEGE
DEPARTMENT OF MATHEMATICS

TEST-2

201-912-DW

Fall 2010 **TIME: 1 H 45 MIN**

Instructor: Gilbert Honnouvo and James Requeima

Name: SOLUTIONS
ID:

Instructions:

- Translation and regular dictionaries are permitted.
- Scientific non-programmable calculators are permitted.
- Print your name and ID in the provided space.

This examination consists of 5 exercises. Please ensure that you have a complete examination before starting.

(1) [*10+10 marks*] In each case, find the components, the magnitude and the standard-position angle of sum R of the given vectors:

(a) $F = 154,\ \theta_F = 90^o$
$T = 128,\ \theta_T = 43^o$

$F_x = F\cos\theta_F = 154\cos 90° = 0,\quad F_y = F\sin\theta_F = 154\sin 90° = 154$

$T_x = T\cos\theta_T = 128\cos 43° = 94\quad T_y = T\sin\theta_T = 128\sin 43° = 87$

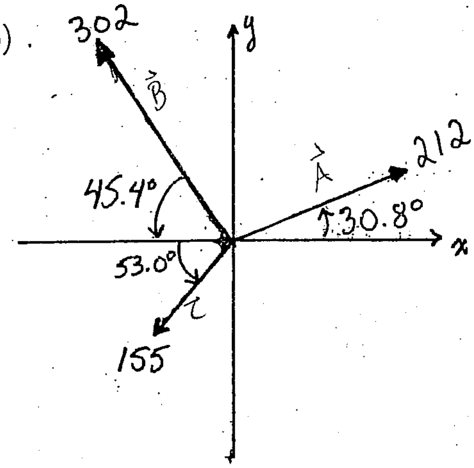
$\therefore\quad R_x = F_x + T_x = 94\qquad R_y = F_y + T_y = 154 + 87 = 241$

$R = \sqrt{94^2 + 241^2} = 259$

$\tan\theta_R = \frac{241}{94}\qquad \tan^{-1}\left(\frac{241}{94}\right) = 69°\qquad \theta_R$ IS IN QUADRANT I

$\therefore\ \boxed{\theta_R = 69°}$

(b)



$A = 212\quad \theta_A = 30.8°$
$B = 302\quad \theta_B = 180° - 45.4° = 134.6°$
$C = 155\quad \theta_C = 180° + 53.0° = 233.0°$

$A_x = 212\cos 30.8° = 182$
$B_x = 302\cos 134.6° = -212$
$B_y = 155\cos 233.0° = -93.3$
$R_x = -123$

$A_y = 212\sin 30.8° = 109$
$B_y = 302\sin 134.6° = 215$
$C_y = 155\sin 233.0° = -124$
$R_y = 200$

$\therefore \quad R = \sqrt{(-123)^2 + (200)^2} = \underline{235}$

$\tan\theta_R = \dfrac{200}{-123} \qquad \tan^{-1}\left(\dfrac{200}{-123}\right) = -58.4^\circ$

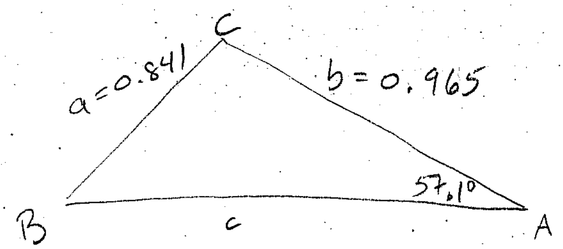
$\therefore \quad \alpha = 58.4^\circ$

BUT θR IS IN QUADRANT II

$\therefore \quad \theta_R = 180 - 58.4^\circ = \underline{121.6^\circ}$

(2) [*10+10 marks*]Solve the triangles with the given parts

(a) $a = 0.841,\ b = 0.965,\ \hat{A} = 57.1^o$



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{b \sin A}{a} = \frac{0.965 \sin 57.1^\circ}{0.841}$$

$$\sin^{-1}\left(\frac{0.965 \sin 57.1^\circ}{0.841}\right) = 74.5^\circ$$

ACUTE CASE

$B = \underline{74.5^\circ}$

$C = 180^\circ - 74.5 - 57.1^\circ$

$= \underline{48.4^\circ}$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{0.841 \sin 48.4^\circ}{\sin 57.1^\circ}$$

$= \underline{0.749}$

OBTUSE CASE

$B = 180^\circ - 74.5^\circ = \underline{105.5^\circ}$

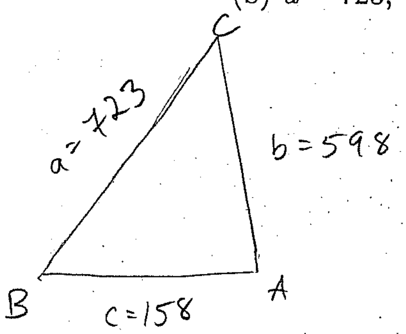
$C = 180^\circ - 105.5^\circ - 57.1^\circ$

$= \underline{17.4^\circ}$

$$c = \frac{a \sin C}{\sin A} = \frac{0.841 \sin 17.4^\circ}{\sin 57.1^\circ}$$

$= \underline{0.300}$

(b) $a = 723,\ b = 598,\ c = 158$



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc} = \frac{723^2 - 598^2 - 158^2}{-2(598)(158)}$$

$$\therefore A = \cos^{-1}\left(\frac{723^2 - 598^2 - 158^2}{-2(598)(158)}\right)$$

$$= \underline{137.9^\circ}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\sin C = \frac{c\sin A}{a} = \frac{158\sin 137.9^\circ}{723}$$

$$C = \sin^{-1}\left(\frac{158\sin 137.9^\circ}{723}\right) = \underline{8.4^\circ}$$

$$B = 180^\circ - 8.4^\circ - 137.9^\circ = \underline{33.7^\circ}$$

(3) [*10+10 marks*]For each function, determine the amplitude, period, the displacement and sketch the graph on any interval with period length.

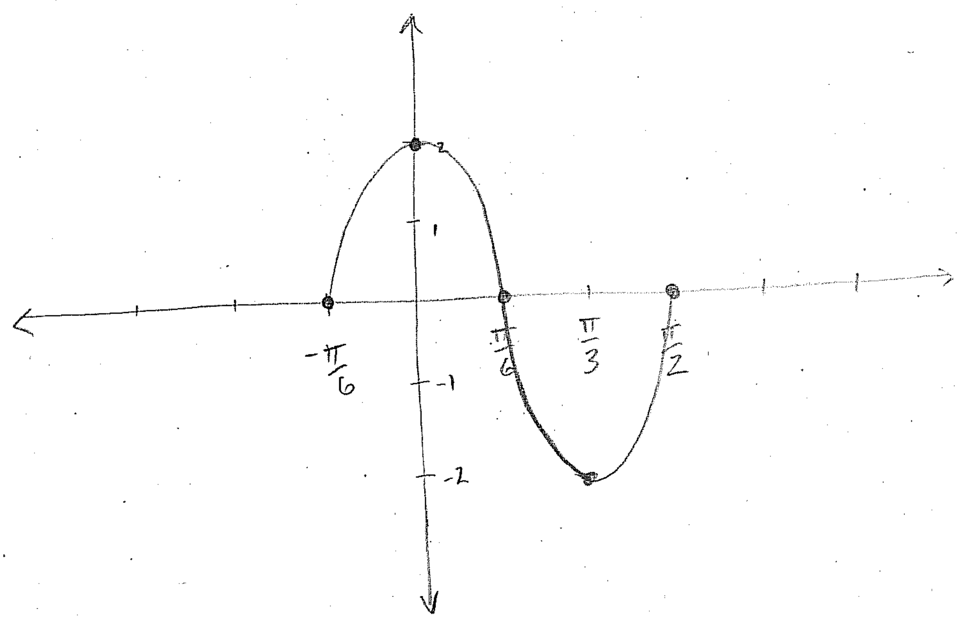
(a) $y = 2\sin\left(3x + \frac{\pi}{2}\right)$

AMPLITUDE : $|a| = |2| = 2$

PERIOD : $\frac{2\pi}{b} = \frac{2\pi}{3} \longrightarrow \frac{2\pi}{3} \div 4 = \frac{\pi}{6}$

DISPLACEMENT $= -\frac{c}{b} = -\frac{\frac{\pi}{2}}{3} = -\frac{\pi}{6}$

x	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	0	2	0	-2	0



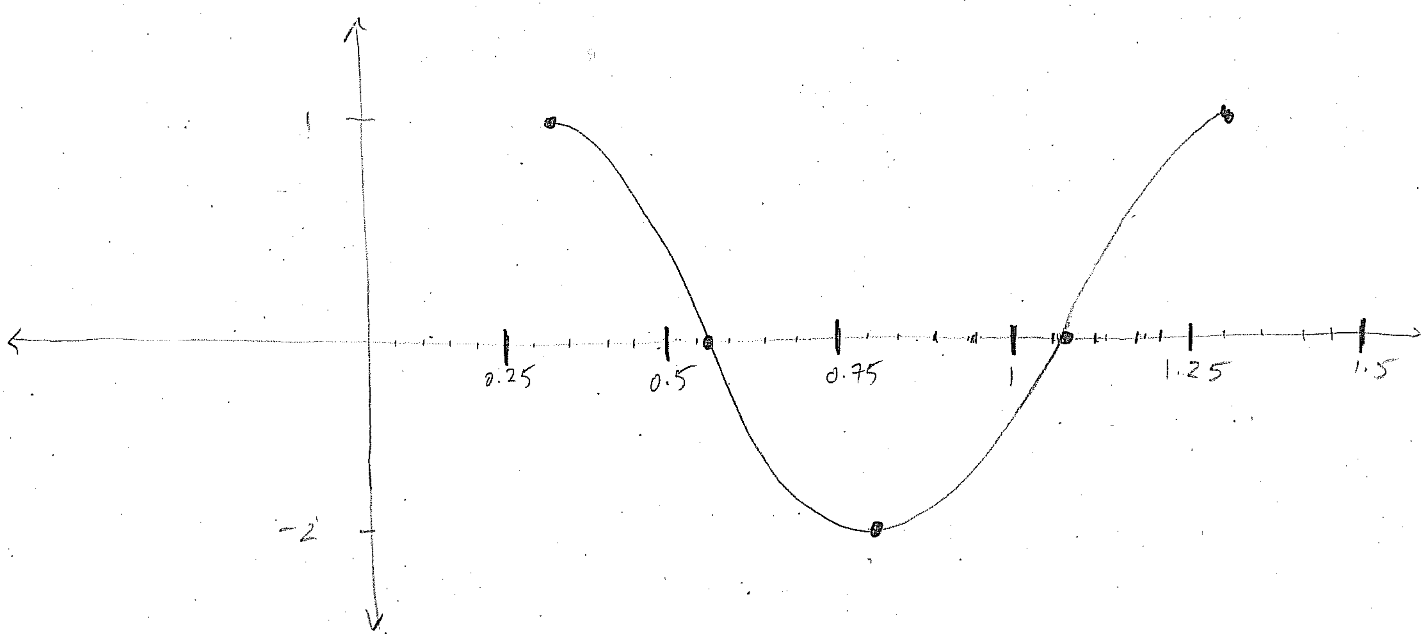
(b) $y = \cos(2\pi x - 2)$

AMPLITUDE: $|a| = |1| = 1$

PERIOD: $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$

DISPLACEMENT: $-\frac{c}{b} = -\frac{(-2)}{2\pi} = \frac{1}{\pi} \approx 0.3183$

x	$\frac{1}{\pi} \approx 0.3183$	$\frac{1}{\pi} + \frac{1}{4} \approx 0.5683$	$\frac{1}{\pi} + \frac{2}{4} \approx 0.8183$	$\frac{1}{\pi} + \frac{3}{4} \approx 1.0683$	$\frac{1}{4} + 1 = 1.3183$
y	1	0	-1	0	1



(4) [*10+10 marks*]Using trigonometric identities, find the EXACT VALUES of the following (no decimal or rounding):

(a) $\sin(105^o)$, $(hint: 105^o = 60^o + 45^o)$

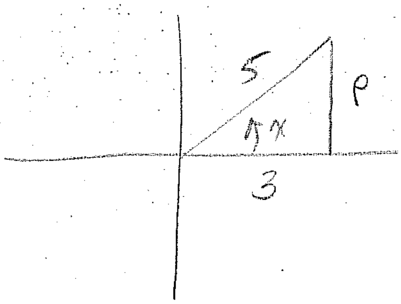
$$\sin(105^\circ) = \sin(60^\circ + 45^\circ)$$

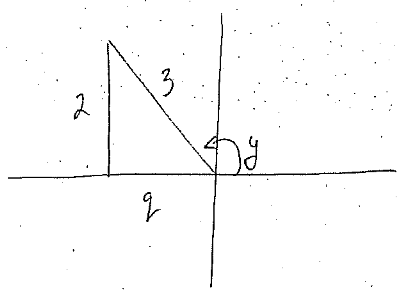
$$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

(b) Let $0 < x < 90^o$ and $90^o < y < 180^o$ such that $\cos(x) = \frac{3}{5}$ and $\sin(y) = \frac{2}{3}$.
Find $\cos(x-y)$





$$p^2 = 5^2 - 3^2$$
$$p = \sqrt{25-9}$$
$$= 4$$

$$\therefore \quad \sin x = \frac{4}{5}$$

$$q^2 = 3^2 - 2^2$$
$$q = -\sqrt{3^2-2^2}$$
$$= -\sqrt{5}$$

$$\therefore \quad \cos y = -\frac{\sqrt{5}}{3}$$

$$\therefore \quad \cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$= \left(\frac{3}{5}\right)\left(\frac{-\sqrt{5}}{3}\right) + \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)$$
$$= \frac{-3\sqrt{5}}{15} + \frac{8}{15}$$
$$= \frac{8-3\sqrt{5}}{15}$$

(5) [*10+10 marks*] Solve the following systems of linear equations:

(a) 3x+17y=7
-10x+13y=5,
using Cramer's Rule

$$x = \frac{\begin{vmatrix} 7 & 17 \\ 5 & 13 \end{vmatrix}}{\begin{vmatrix} 3 & 17 \\ -10 & 13 \end{vmatrix}} = \frac{(7)(13)-(17)(5)}{(3)(13)-(17)(-10)} = \frac{6}{209}$$

$$y = \frac{\begin{vmatrix} 3 & 7 \\ -10 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 17 \\ -10 & 13 \end{vmatrix}} = \frac{(3)(5)-(7)(-10)}{209} = \frac{85}{209}$$

(b) $2x+4y=-3$ (1)
$7x+3y=10,$ (2)
using substitution method.

(1) $2x = -3 - 4y$

$x = -\frac{3}{2} - 2y$

(2) $7x + 3y = 10$

$7\left(-\frac{3}{2} - 2y\right) + 3y = 10$

$-\frac{21}{2} - 14y + 3y = 10$

$-11y = 10 + \frac{21}{2}$

$-11y = \frac{41}{2}$

$y = -\frac{41}{22}$

$x = -\frac{3}{2} - 2\left(-\frac{41}{22}\right)$

$= -\frac{3}{2} + \frac{41}{11}$

$= -\frac{33}{22} + \frac{82}{22}$

$= \frac{49}{22}$

$\therefore x = \frac{49}{22}$ $\quad y = -\frac{41}{22}$

FORMULA SHEET

$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$
$\sin(x-y) = \sin(x)\cos(y) - \sin(y)\cos(x)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

For a vector A in xy-coordinate plane, the x component A_x, the y component A_y the magnitude and the reference angle are given by:

$A_x = A\cos(\theta)$, $A_y = A\sin(\theta)$
$A = \sqrt{A_x^2 + A_y^2}$
$\theta_{ref} = \tan^{-1}\left(\left|\frac{A_y}{A_x}\right|\right)$

Law of sine

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of cosine

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$