Dawson College: Civil Technology: 201-912-DW

November 25, 2010

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 3

The length of the test is 1hr and 45min.

Question 1. *(5 marks)* Find the x and y-intercepts and vertex of $y = -5x^2 + 7x + 13$. Graph this parabola.

y-int: $x = 0$

$y = -5(0)^2 + 7(0) + 13$
$= 13$

$\therefore (0, 13)$

x-int: $y = 0$

$0 = -5x^2 + 7x + 13$

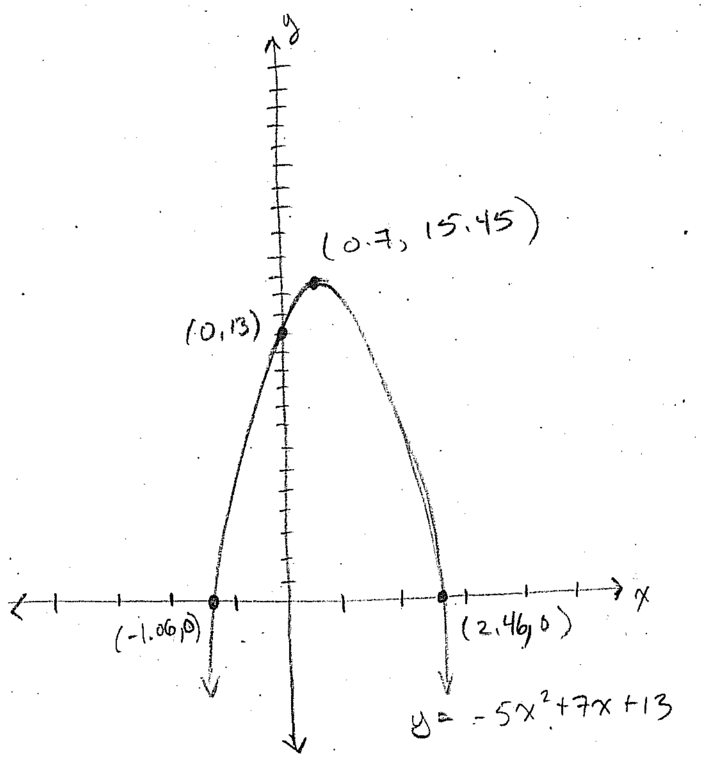
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-7 \pm \sqrt{7^2 - 4(-5)(13)}}{2(-5)}$

$= \dfrac{-7 \pm \sqrt{309}}{-10} = -1.058$ AND 2.458

$\therefore (-1.06, 0), (2.46, 0)$

$x_v = \dfrac{-b}{2a} = \dfrac{-7}{2(-5)} = 0.7$

$y_v = -5(0.7)^2 + 7(0.7) + 13$
$= 15.45$



Question 2. *(10 marks)* Use Cramer's rule to find the equation of the quadratic function that passes through the points $(1,2)$, $(3,14)$, and $(-2,29)$.

$y = ax^2 + bx + c$

$2 = a(1)^2 + b(1) + c \qquad 14 = a(3)^2 + b(3) + c \qquad 29 = a(-2)^2 + b(-2) + c$

① $2 = a + b + c \qquad$ ② $14 = 9a + 3b + c \qquad$ ③ $29 = 4a - 2b + c$

$$a = \frac{\left|\begin{array}{ccc} 2 & 1 & 1 \\ 14 & 3 & 1 \\ 29 & -2 & 1 \end{array}\right| \begin{array}{cc} 2 & 1 \\ 14 & 3 \\ 29 & -2 \end{array}}{\left|\begin{array}{ccc} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 4 & -2 & 1 \end{array}\right| \begin{array}{cc} 1 & 1 \\ 9 & 3 \\ 4 & -2 \end{array}} = \frac{6 + 29 + (-28) - 87 - (-4) - 14}{3 + 4 + (-18) - 12 - (-2) - (9)} = \frac{-90}{-30}$$

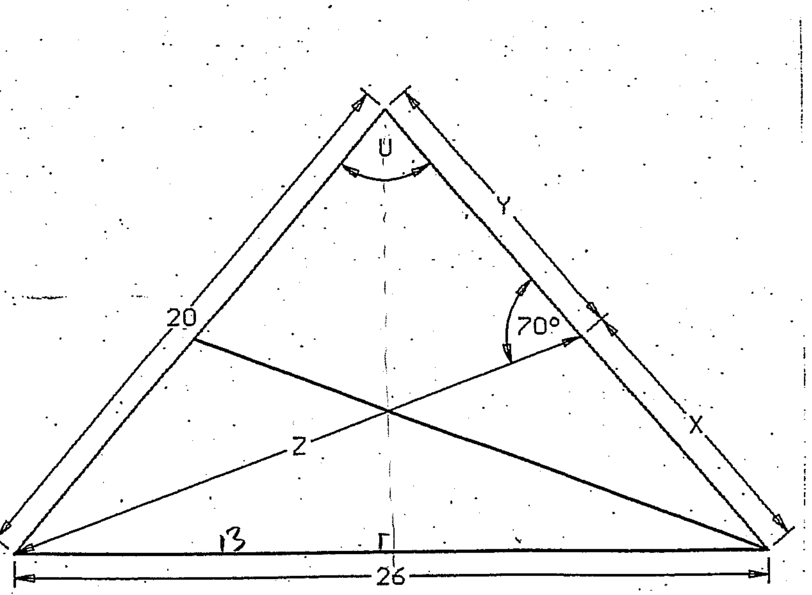
$$= 3$$

$$b = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 9 & 14 & 1 \\ 4 & 29 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 4 & -2 & 1 \end{vmatrix}} = \frac{180}{-30} = -6$$

$$c = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 9 & 3 & 14 \\ 4 & -2 & 29 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 4 & -2 & 1 \end{vmatrix}} = \frac{-150}{-30} = 5$$

$\therefore \; y = 3x^2 - 6x + 5$

Question 3. *(10 marks)* For the following symmetric scissortruss find X, Y, Z and U. All lengths are in metres.



$$\sin\left(\tfrac{1}{2}u\right) = \frac{13}{20} \Rightarrow \tfrac{1}{2}u = \sin^{-1}\left(\frac{13}{20}\right) = 40.54160187^\circ$$

$$\therefore \quad u = 81.08320375^\circ$$

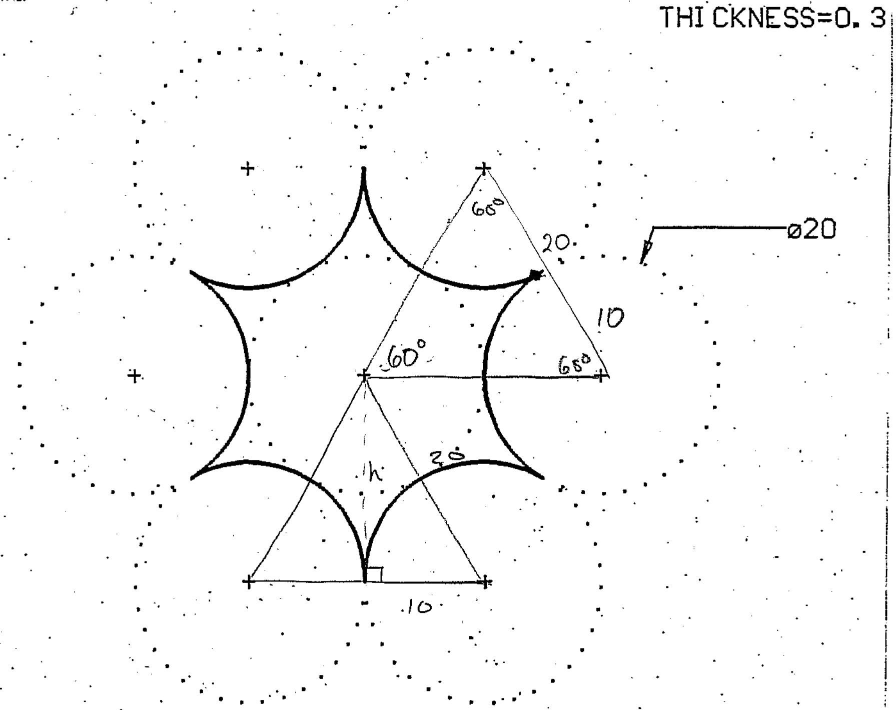
$$\frac{20}{\sin 70^\circ} = \frac{z}{\sin u} \Rightarrow z = \frac{20(\sin 81.08320375^\circ)}{\sin 70^\circ} = 21.0263324\text{m}$$

$$\theta = 180^\circ - 70^\circ - u = 28.91679625^\circ$$

$$\frac{y}{\sin\theta} = \frac{20}{\sin 70^\circ} \Rightarrow y = \frac{20\sin\theta}{\sin 70^\circ} = 10.29142922\text{ m}$$

$$x = 20 - y = 20 - 10.29142922 = 9.70857077\text{9m}$$

Question 4. *(10 marks)* For the following floorplan find the curcumference, area and volume. All lengths are in metres.



$\theta = \frac{360}{6} = 60$

$\therefore$ CIRCUMFERENCE $= 12\left(60° \cdot \frac{\pi}{180°} \cdot 10\right) = 125.6637061\text{m}$

$h = \sqrt{20^2 - 10^2} = 17.32050808$

$\therefore$ AREA $= 6$ (AREA OF Δ) $- 12$ (AREA OF SECTOR)

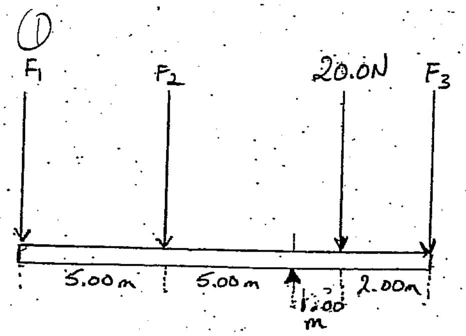
$= 6\left(\frac{1}{2}(20)(17.32050808)\right) - 12\left(\frac{1}{2}\,60°\left(\frac{\pi}{180°}\right)(10)^2\right)$

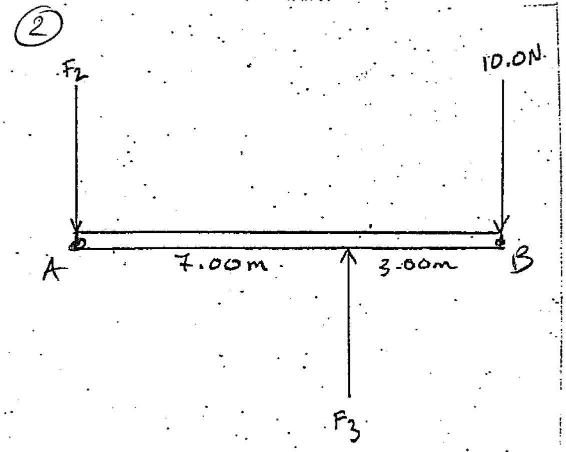
$= 1039.230485 - 628.3185307$

$= 410.9119543\text{ m}^2$

VOLUME $= (410.9114543)(0.3) = 123.2735863\text{m}^3$

Question 5. *(10 marks)* The systems below are in equilibrium. Find F_1, F_2 and F_3.





FROM 2!

@ A $\overset{\curvearrowright}{\Sigma M} = \overset{\curvearrowleft}{\Sigma M}$

$(10.00)(10.0) = (7.00) F_3$

$\therefore F_3 = 14.3\,N$

@ B $\overset{\curvearrowright}{\Sigma M} = \overset{\curvearrowleft}{\Sigma M}$

$3 F_3 = 10 F_2$

$\frac{3(14.3)}{10} = F_2$

$\therefore F_2 = 4.29\,N$

FROM 1!

$\overset{\curvearrowright}{\Sigma M} = \overset{\curvearrowleft}{\Sigma M}$

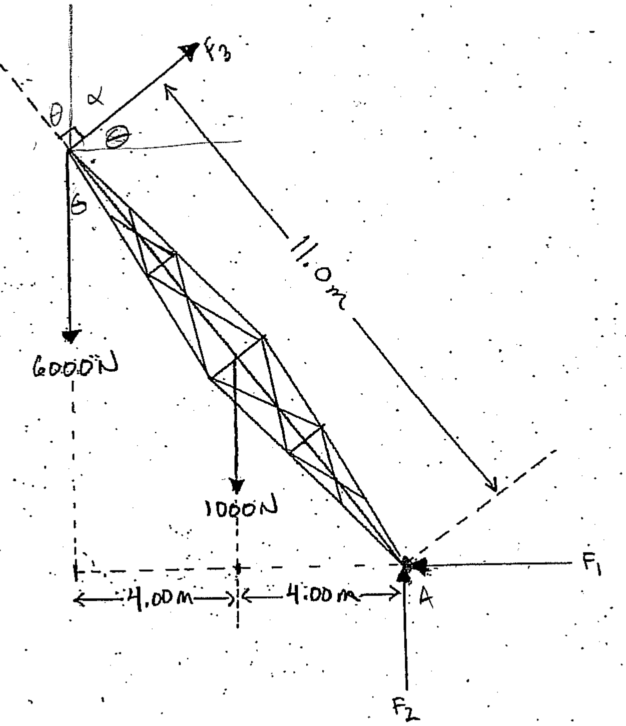
$10F_1 + 5F_2 = (1)(20.0) + 3F_3$

$10 F_1 = 20.0 + 3(14.3) - 5(4.29)$

$10 F_1 = 41.45$

$F_1 = 4.15\,N$

Question 6. *(10 marks)* Find F_1, F_2 and F_3 in the following diagram of a crain given that the system is in equilibrium.



@ A

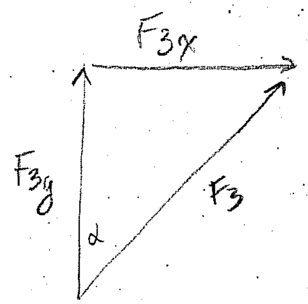
$\overset{\curvearrowright}{OM} = \overset{\curvearrowleft}{OM}$

$11\, F_3 = (6000)(8.00) + (1000)(4.00)$

$F_3 = \underline{4727\ N}$

Now

$\sin\theta = \frac{8}{11} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{11}\right) = 46.66°$



$\alpha = 90° - 46.66°$
$= 43.34°$

$\therefore \sin\alpha = \frac{F_{3x}}{F_3} \Rightarrow F_{3x} = 4727 \sin 43.34° = 3244\ N$

$\cos\alpha = \frac{F_{3y}}{F_3} \Rightarrow F_{3y} = 4727 \cos 43.34° = 3438\ N$

$\therefore \Sigma x: \quad F_1 = F_{3x} = \underline{3244\ N}$

$\Sigma y: \quad F_2 + F_{3y} = 6000 + 1000$

$F_2 = 7000 - 3438 = 3562\ N$

Question 7. *(5 marks)* Solve the system:

$$\begin{array}{llcrcl} ① & 2y^2 & - & 4x & = & 7 \\ ② & y^2 & + & 2x^2 & = & 3 \end{array}$$

① $2y^2 - 4x = 7$

② $\times 2 - (2y^2 + 4x^2 = 6)$

$-4x^2 - 4x = 1$

$0 = 4x^2 + 4x + 1$

$0 = (2x+1)^2$

$x = -1/2$

$2y^2 - 4(-1/2) = 7$

$2y^2 = 7 - 2$

$2y^2 = 5$

$y^2 = 5/2$

$y = \pm\sqrt{5/2}$

$\therefore \left(-\frac{1}{2}, \sqrt{5/2}\right)$ AND $\left(1/2, -\sqrt{5/2}\right)$ ARE THE SOLUTIONS.