DAWSON COLLEGE
Mathematics Department
Final Examination
Calculus I 201-NYA-05
(Commerce and International Business Studies)
December 18th, 2008

1. (5 Marks) Find the value of the constant k so that function $f(x)$ will be continuous at $x=2$. Verify the conditions of continuity for $f(x)$ at $x=2$.

$$f(x)=\begin{cases} 5x^3-40 & \text{if } x>2 \\ k+6 & \text{if } x=2 \\ x-2 & \text{if } x<2 \end{cases}$$

2. (2+3+3 Marks) Evaluate the following limits, if possible

a) $\lim_{x\to 2}\dfrac{2x^2+7x-18}{x^2-4x}$ b) $\lim_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2}}{x}$ c) $\lim_{x\to 3}\dfrac{4x-12}{\frac{1}{x+1}-\frac{1}{4}}$

3. (4 Marks) Use **the limit definition of the derivative** to find $\dfrac{dy}{dx}$ if $y=x^2+3x+1$.

4. (4 Marks) Find an equation for the line tangent to $y=\dfrac{1-x}{1+x}$ at $x=2$.

5. (5 Marks) Find the x-value(s) at which the graph of the function $y=\dfrac{(3x+1)^2}{(7x+2)^3}$ has a horizontal tangent line.

6. (16 Marks) Differentiate the following functions (make only obvious simplifications)

a) $y=\arctan\left(4x^3\right)$ b) $y=\dfrac{\tan(5x)}{\left(x^2+7x\right)^3}$ c) $y=e^{5x^2}\cdot\cos(2x)$ d) $y=\sin^2(3x)$

7. (5 Marks) Use implicit differentiation to find $\dfrac{dy}{dx}$ if $x^2+2xy+y^3=5x$.

8. (5 Marks) Find $\dfrac{d^2y}{dx^2}$ and simplify your answer if $y=\ln\left(1+x^4\right)$.

9. (5 Marks) Use logarithmic differentiation method to find the derivative of $y=\dfrac{\left(x^2+1\right)^{\frac{1}{4}}\cdot\sqrt{\sin x}}{e^{2x}\left(x^2+3x+1\right)^5}$.

10. (6 Marks) The temperature of a cup of coffee at time t (in minutes) is $T(t)=70+C\cdot e^{-0.06t}$. Initially (i.e. at time $t=0$) the temperature of the coffee was $200^\circ F$.
    a) Find the constant C.
    b) When will the temperature of the coffee be $150^\circ F$? (Round off to two decimal places.)
    c) How fast is the coffee temperature decreasing at $t=4$ minutes? (Round off to two decimal places.)

11. (6 Marks) The demand function for a certain product is $x = f(p) = \sqrt{450-5p}$ $(0 \le p \le 90)$.

a) Find the elasticity of demand. (Recall that the elasticity of demand can be expressed in the form $E = -\frac{p \cdot f'(p)}{f(p)}$ or $E = -\frac{p}{x}\left(\frac{dx}{dp}\right)$)

b) Is the demand elastic or inelastic when (i) $p = 40$ (ii) $p = 70$?

c) When is the demand unitary? (Determine the price.)

12. (6 Marks) The demand function for a certain product is $p = \frac{50}{0.01x^2+1}$ $(0 \le x \le 20)$ where p is in dollars and x is measured in units. Find

a) Revenue function $R(x)$.

b) Marginal revenue function $R'(x)$.

c) $R'(2)$ and interpret your result.

13. (6 Marks) The total cost for a company to produce x units of a good is given by $C(x) = 0.02x^2 + 30x + 72$

a) Find the average cost function.

b) Find the level of production that minimizes the average cost.

14. (6 Marks) A book designer requires the pages of a book to have 2 cm margins at the top and bottom and 1 cm margins on the sides. Furthermore, page area is required to be 200 cm^2. Find the dimensions of the page that will result in the maximum printed area on the page.

15. (8 Marks) Given the function $f(x) = x^3 - 9x^2 + 15x - 5$.

Find (if any)

a) the y-intercept

b) the intervals where $f(x)$ is increasing and where it is decreasing

c) all relative (local) maxima and relative (local) minima .

d) the intervals where $f(x)$ is concave upward and where it is concave downward

e) all points of inflection

Using this information **sketch the graph of** $f(x)$. **Clearly label all the points found**

16. (5 Marks) Find the indefinite integral $\int\left(\frac{2}{x^4} - 5\sec^2 x + \frac{7}{x}\right) dx$

ANSWERS

1. $k=-6$

2. a) -1 b) $\frac{1}{2\sqrt{2}}$ c) -64

3. $2x+3$

4. $y=\frac{-2}{9}x+\frac{1}{9}$ or $9y+2x-1=0$

5. $x=-\frac{1}{3}, x=-\frac{3}{7}$

6. a) $\frac{12x^2}{1+16x^6}$ b) $\frac{5\sec^2(5x)\cdot(x^2+7x)-3\tan(5x)(2x+7)}{(x^2+7x)^4}$

   c) $2e^{5x^2}\cdot[5x\cdot\cos(2x)-\sin(2x)]$ d) $6\sin(3x)\cos(3x)$

7. $y'=\frac{5-2x-2y}{2x+3y^2}$

8. $\frac{d^2y}{dx^2}=\frac{4x^2(3-x^4)}{(1+x^4)^2}$

9. $y'=\frac{(x^2+1)^{\frac{1}{4}}\cdot\sqrt{\sin x}}{e^{2x}(x^2+3x+1)^5}\cdot\left[\frac{x}{2(x^2+1)}+\frac{1}{2}\cot x-2-\frac{5(2x+3)}{x^2+3x+1}\right]$

10. a) $C=130$ b) $t=-\frac{1}{0.06}\ln\left(\frac{8}{13}\right)\approx 8.09\text{ min}$ c) $\left.\frac{dT}{dt}\right|_{t=4}=-7.8e^{-0.24}\approx -6.14\ {}^\circ F/\text{min}$

11. a) $E(p)=\frac{p}{2(90-p)}$ b) (i) inelastic (ii) elastic c) \$60.

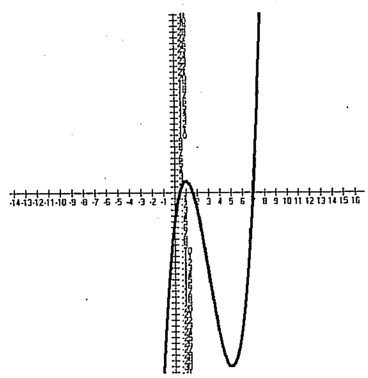
12. a) $R(x)=\frac{50x}{0.01x^2+1}$ b) $R'(x)=\frac{50-0.5x^2}{(0.01x^2+1)^2}$ c) $R'(2)=\frac{48}{1.04^2}\approx 44.38$

The sale of the 3rd unit will bring a revenue of approximately \$44.38

13. a) $\overline{C}(x)=0.02x+30+\frac{72}{x}$ b) 60 units.

14. 10 cm by 20 cm.

15.
   a) (0,-5)
   b) $f(x)$ is increasing on $(-\infty,1)\cup(5,\infty)$ and $f(x)$ is decreasing on $(1,5)$
   c) a relative (local) maximum is at (1,2), a relative (local) minimum is at (5,-30)
   d) $f(x)$ is concave upward on $(3,\infty)$ and it is concave downward on $(-\infty,3)$
   e) (3,-14) is an inflection point



16. $\frac{2x^{-3}}{-3}-5\tan x+7\ln|x|+c$