

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 1 (B)

Question 1. (3 marks) Rationalize the denominator in the expression:

$$\frac{y}{\sqrt[3]{z^2x}} = \frac{y}{(z^2x)^{1/3}} \cdot \frac{(z^2x)^{2/3}}{(z^2x)^{2/3}} = \frac{y z^{4/3} x^{2/3}}{z^2 x}$$

Question 2. (3 marks) Simplify the expression (express your answer with positive exponents only):

$$\begin{aligned} \left(\frac{a^3}{-27b^{-6}}\right)^{-2/3} &= \frac{a^{-2}}{(-27)^{-2/3} b^4} = \frac{(-27)^{2/3}}{a^2 b^4} \\ &= \frac{(\sqrt[3]{-27})^2}{a^2 b^4} = \frac{9}{a^2 b^4} \end{aligned}$$

Question 3. (6 marks) Factor the following completely:

$$\begin{aligned} \text{(a) } x^3 - 3x^2 - 4x + 12 &= x^2(x-3) - 4(x-3) \\ &= (x-3)(x^2-4) \\ &= (x-3)(x+2)(x-2) \end{aligned}$$

$$\begin{aligned}
 & \text{(b) } 12x^2 - 23x + 10 \\
 & = 12x^2 - 15x - 8x + 10 \\
 & = 3x(4x - 5) - 2(4x - 5) \\
 & = (4x - 5)(3x - 2)
 \end{aligned}$$

$$\begin{aligned}
 \alpha \cdot \beta &= 120 \\
 \alpha + \beta &= -23 \\
 \alpha &= -15 \quad \beta = -8
 \end{aligned}$$

$$\begin{aligned}
 & \text{(c) } (x-1)^3 + 8 \\
 & = [(x-1) + 2][x^2 - 2(x-1) + 2^2] \\
 & = (x+1)(x^2 - 2x + 1 - 2x + 2 + 4) \\
 & = (x+1)(x^2 - 4x + 7)
 \end{aligned}$$

Question 4. (8 marks) Simplify the following expressions:

$$\begin{aligned}
 \text{(a) } \frac{y+z^2y^{-1}}{y-z^4y^{-3}} &= \frac{y + \frac{z^2}{y}}{y - \frac{z^4}{y^3}} = \frac{\frac{y^2}{y} + \frac{z^2}{y}}{\frac{y^4}{y^3} - \frac{z^4}{y^3}} = \frac{\frac{y^2 + z^2}{y}}{\frac{y^4 - z^4}{y^3}} \\
 &= \frac{y^2 + z^2}{y} \cdot \frac{y^3}{y^4 - z^4} = \frac{y^2(y^2 + z^2)}{(y^2 + z^2)(y^2 - z^2)} = \frac{y^2}{y^2 - z^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \frac{1}{1 - \frac{1}{1+a}} + \frac{1}{1 - \frac{1}{1-a}} &= \frac{1}{\frac{1+a}{1+a} - \frac{1}{1+a}} + \frac{1}{\frac{1-a}{1-a} - \frac{1}{1-a}} \\
 &= \frac{1}{\frac{a}{1+a}} + \frac{1}{\frac{-a}{1-a}} = \frac{1+a}{a} + \frac{1-a}{-a} = \frac{1+a}{a} - \frac{1-a}{a} \\
 &= \frac{(1+a) - (1-a)}{a} = \frac{2a}{a} = 2
 \end{aligned}$$