

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 2 (A)

Question 1. (5 marks) Find the first and second derivative of the function:

$$f(t) = \frac{t}{t^2-1}, \quad f'(t) = \frac{(1)(t^2-1) - t(2t)}{(t^2-1)^2} = \frac{-t^2-1}{(t^2-1)^2}$$

$$\begin{aligned} f''(t) &= \frac{(-2t)(t^2-1)^2 - (-t^2-1)[2(t^2-1)(2t)]}{(t^2-1)^4} \\ &= \frac{(2t)(t^2-1)[- (t^2-1) - (-t^2-1)(2)]}{(t^2-1)^4} \\ &= \frac{(2t)(t^2-1)(t^2+3)}{(t^2-1)^3} = \frac{2t(t^2+3)}{(t^2-1)^3} \end{aligned}$$

Question 2. (5 marks) Find $\frac{dy}{dx}$:

$$y^{1/2}x^2 = x + 2y^3$$

$$\frac{d}{dx}(y^{1/2}x^2) = \frac{d}{dx}(x) + \frac{d}{dx}(2y^3)$$

$$\frac{1}{2}y^{-1/2}y'x^2 + y^{1/2} \cdot 2x = 1 + 6y^2y'$$

$$y' \frac{x^2}{2y^{1/2}} - 6y^2y' = 1 - 2y^{1/2}x$$

$$y' \left(\frac{x^2}{2y^{1/2}} - 6y^2 \right) = 1 - 2y^{1/2}x$$

$$y' \left(\frac{x^2 - 12y^{5/2}}{2y^{1/2}} \right) = 1 - 2y^{1/2}x$$

$$y' = \frac{(1 - 2y^{1/2}x)(2y^{1/2})}{x^2 - 12y^{5/2}}$$

Question 3. (5 marks) Find the intervals where the function is increasing, the intervals where the function is decreasing and any relative extrema:

$$f(x) = \frac{10}{3}x^3 - 10x^2 - 30x - 10$$

$$f'(x) = 10x^2 - 20x - 30$$

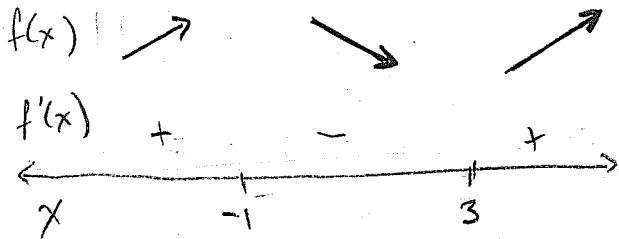
$$f'(x) = 0$$

$$10x^2 - 20x - 30 = 0$$

$$10(x^2 - 2x - 3) = 0$$

$$10(x - 3)(x + 1) = 0$$

$$x = -1, 3$$



TEST POINTS:

$$x = -2: f'(-2) = 10(-2)^2 - 20(-2) - 30 = 40 + 40 - 30 = 50 > 0$$

$$x = 0: f'(0) = -30 < 0$$

$$x = 4: f'(4) = 10(4)^2 - 20(4) - 30 = 50 > 0$$

f IS INCREASING ON $(-\infty, -1)$ AND $(3, \infty)$

f IS DECREASING ON $(-1, 3)$.

$$\therefore f(-1) = \frac{10}{3}(-1)^3 - 10(-1)^2 - 30(-1) - 10 = \frac{20}{3} \text{ IS A}$$

RELATIVE MAXIMUM

$$f(3) = \frac{10}{3}(3)^3 - 10(3)^2 - 30(3) - 10 = -100 \text{ IS A}$$

RELATIVE MINIMUM